

Collectivity from covariant transport

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- what is covariant transport theory
- what it tells us about collectivity at RHIC
 - cooling, v_2 , heavy quarks, soft/hard physics boundary, HBT
- many puzzles, open questions

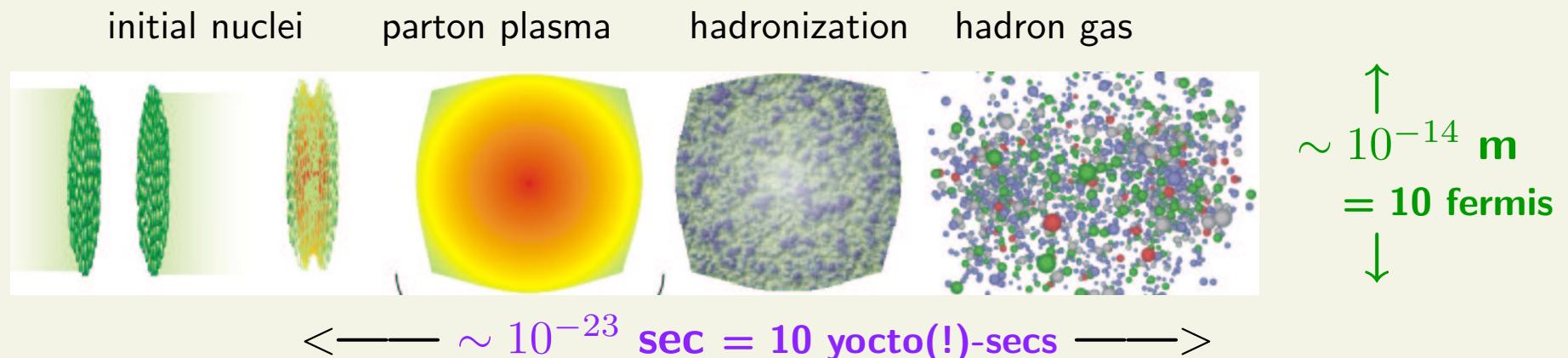
Heavy-ion physics

- partonic condensed matter physics Kajantie '96

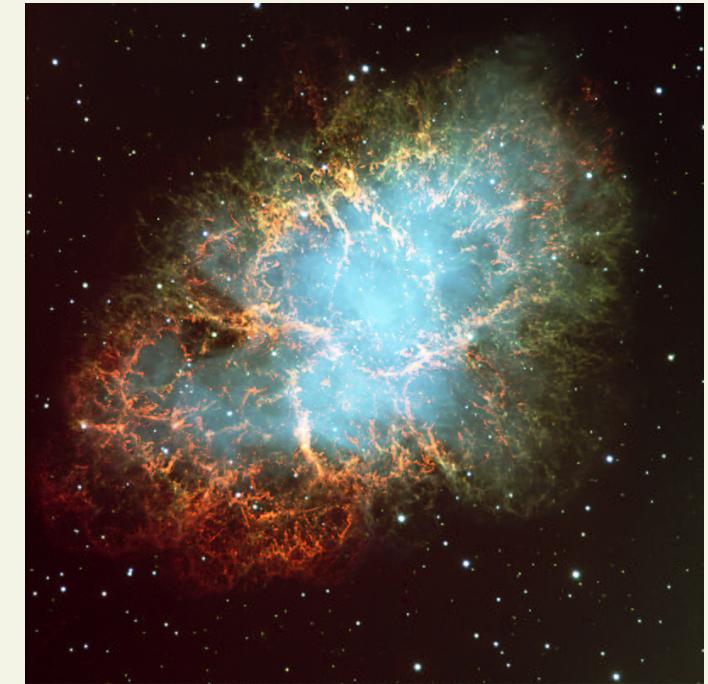
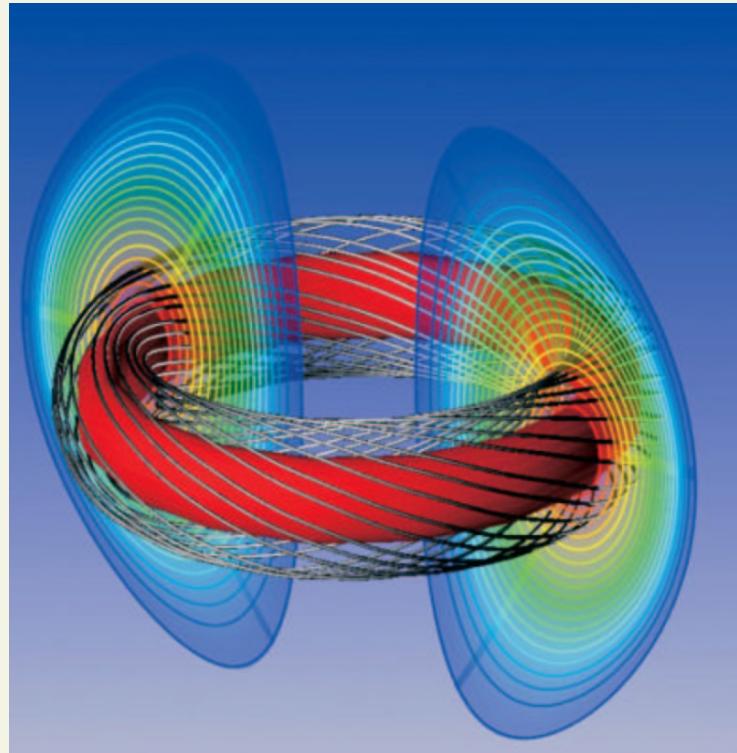
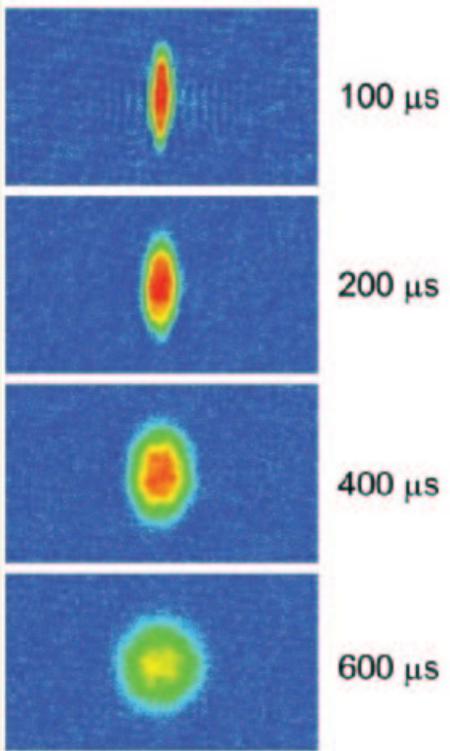
many-body system $\gg \sum$ constituents

- collision dynamics (\sim plasma physics, nonlinear systems)

evolving system \gg system in a box



Exciting commonalities with



strongly-coupled cold atoms - E&M plasmas

- supernovas

Dynamical frameworks

- **hydrodynamics** Csernai, Stöcker, Rischke, Shuryak, Teaney, Heinz, Kolb, Huovinen, Hirano, Muronga, ...
includes phase transitions
but limited to equilibrium, decoupling problem

Euler (ideal) hydro, Navier-Stokes (viscous hydro)

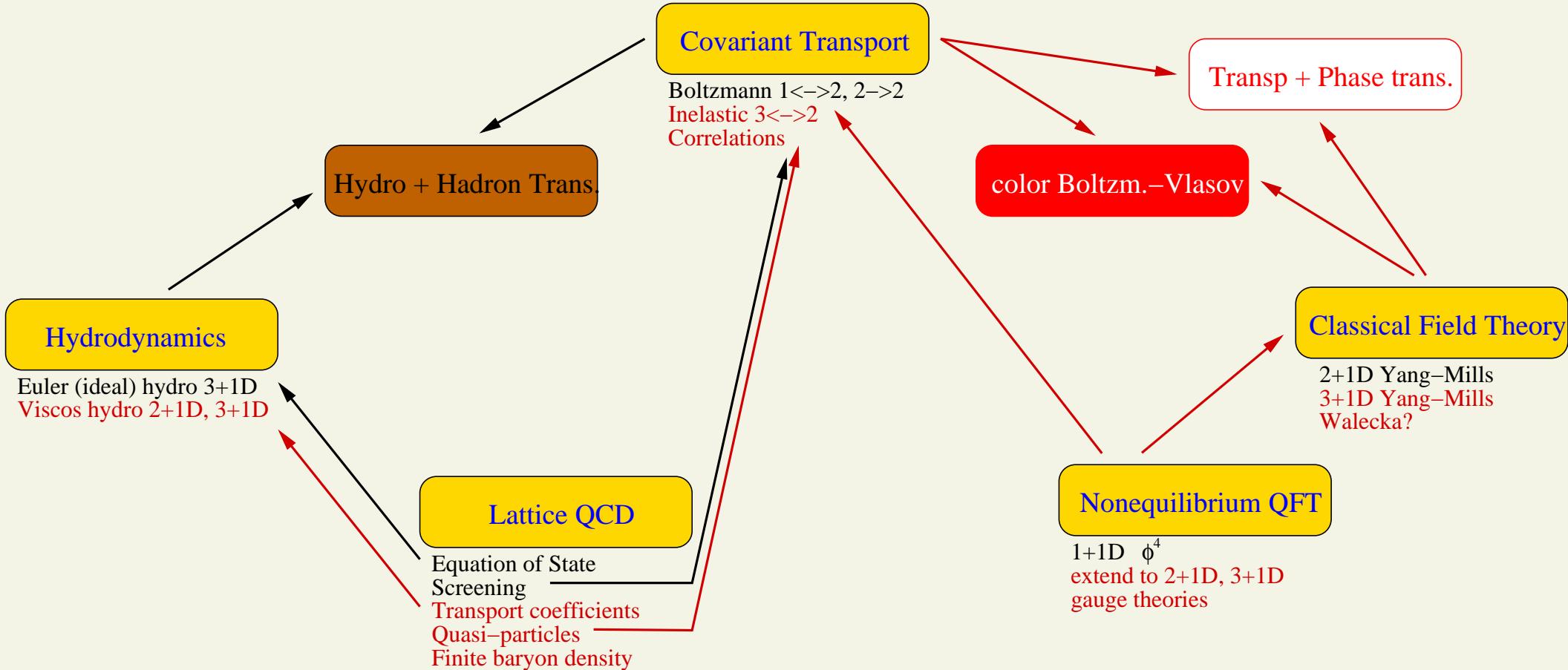
- **covariant transport** Elze, Gyulassy, Heinz, Pang, Zhang, Vance, DM, Csizmadia, Pratt, Cheng, Greiner, Xu
completely non-equilibrium, self-consistent freezeout
but no phase transitions, no coherence effects

parton cascade, hadron cascade

- **classical field theory** Venugopalan, McLerran, Rischke, Krasnitz, Nara, Lappi, ...
has all wave phenomena
but short-wavelength sector problematic

classical Yang-Mills (color glass)

Bigger picture



Covariant transport theory

S. R. de Groot, *Relativistic Kinetic Theory - Principles and Applications*

R. L. Liboff, *Kinetic Theory: Classical, Quantum, and Relativistic Descriptions*

Elze, Gyulassy & Vasak, PLB 177 ('86) 402; Elze & Heinz, Phys. Rep. 183 ('89) 81 + [Ph.D. theses](#)

local scattering rate

$$\frac{dN_{sc}}{d^3x dt} = n_{target} \cdot j_{beam} \cdot \sigma = n_{target}(\vec{x}, t) n_{beam}(\vec{x}, t) v_{rel} \sigma$$

consider also momenta: $n(\vec{x}, t) \rightarrow f(\vec{x}, \vec{p}, t) \equiv dN/d^3x d^3p$

$$\begin{aligned} \frac{\partial f_b(\vec{x}, \vec{p}, t)}{\partial t} = & - \int f_t(\vec{x}, \vec{p}_1, t) f_b(\vec{x}, \vec{p}, t) v_{rel}(\vec{p}, \vec{p}_1) \frac{d\sigma(p, p_1 \rightarrow p', p'_1)}{d^3p' d^3p'_1} d^3p' d^3p'_1 d^3p_1 \\ & + \int f_t(\vec{x}, \vec{p}', t) f_b(\vec{x}, \vec{p}'_1, t) v_{rel}(\vec{p}', \vec{p}'_1) \frac{d\sigma(p', p'_1 \rightarrow p, p_1)}{d^3p d^3p_1} d^3p_1 d^3p'_1 d^3p' \end{aligned}$$

and free streaming: $f(\vec{x}, \vec{p}, t) = f(\vec{x} + \vec{v}\Delta t, \vec{p}, t + \Delta t)$

⇒ **Boltzmann eq:** $p^\mu \partial_\mu f(x, \vec{p}) = C[f]$

Example: Molnar's Parton Cascade

Elementary processes: elastic $2 \rightarrow 2$ processes + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \rightarrow q'\bar{q}'$ + $ggg \leftrightarrow gg$

Equation for $f^i(x, \vec{p})$: $i = \{g, d, \bar{d}, u, \bar{u}, \dots\}$

$$\begin{aligned}
 p_1^\mu \partial_\mu \tilde{\mathbf{f}}^i(x, \vec{p}_1) &= \frac{\pi^4}{2} \sum_{jkl} \int_2 \int_3 \int_4 \left(\tilde{\mathbf{f}}_3^k \tilde{\mathbf{f}}_4^l - \tilde{\mathbf{f}}_1^i \tilde{\mathbf{f}}_2^j \right) \left| \bar{\mathcal{M}}_{12 \rightarrow 34}^{i+j \rightarrow k+l} \right|^2 \delta^4(12 - 34) \xrightarrow{2 \rightarrow 2} \\
 &+ \frac{\pi^4}{12} \int_2 \int_3 \int_4 \int_5 \left(\frac{\tilde{\mathbf{f}}_3^i \tilde{\mathbf{f}}_4^i \tilde{\mathbf{f}}_5^i}{g_i} - \tilde{\mathbf{f}}_1^i \tilde{\mathbf{f}}_2^i \right) \left| \bar{\mathcal{M}}_{12 \rightarrow 345}^{i+i \rightarrow i+i+i} \right|^2 \delta^4(12 - 345) \xrightarrow{2 \leftrightarrow 3} \\
 &+ \frac{\pi^4}{8} \int_2 \int_3 \int_4 \int_5 \left(\tilde{\mathbf{f}}_4^i \tilde{\mathbf{f}}_5^i - \frac{\tilde{\mathbf{f}}_1^i \tilde{\mathbf{f}}_2^i \tilde{\mathbf{f}}_3^i}{g_i} \right) \left| \bar{\mathcal{M}}_{45 \rightarrow 123}^{i+i \rightarrow i+i+i} \right|^2 \delta^4(123 - 45) \xrightarrow{3 \leftrightarrow 2} \\
 &+ \tilde{\mathcal{S}}^i(x, \vec{p}_1) \xleftarrow{\text{initial conditions}}
 \end{aligned}$$

with shorthands:

$$\tilde{\mathbf{f}}_i^q \equiv (2\pi)^3 f_q(x, \vec{p}_i), \quad \int_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \quad \delta^4(p_1 + p_2 - p_3 - p_4) \equiv \delta^4(12 - 34)$$

Nonlinear 6+1D transport eqn: solvable numerically

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = \overbrace{S_i(\vec{x}, \vec{p}, t)}^{\text{source } 2 \rightarrow 2 \text{ (ZPC, GCP, ...)}} + \overbrace{C_i^{el.}[f](\vec{x}, \vec{p}, t)}^{2 \leftrightarrow 3 \text{ (MPC)}} + \overbrace{C_i^{inel.}[f](\vec{x}, \vec{p}, t)} + \dots$$

highly relativistic case → few **covariant/causal algorithms**: ZPC, MPC, Bjorken- τ , ...
algorithms → **cascade** Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, ...
→ **recent attempt - spatial grid** Greiner, Xu ...

code repository @ <http://nt3.phys.columbia.edu/OSCAR>

mean free path:

$$\lambda \equiv \frac{1}{\text{cross section} \times \text{density}} \quad \left\{ \begin{array}{l} \lambda = 0 \text{ -- ideal hydrodynamics} \\ \lambda = \infty \text{ -- free streaming} \end{array} \right.$$

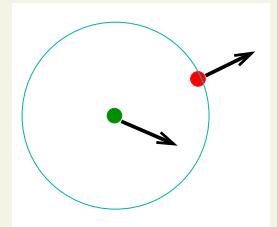
transport opacity: DM & Gyulassy NPA 697 ('02)

$$\chi \equiv \langle n_{coll} \rangle \underbrace{\langle \sin^2 \theta_{CM} \rangle}_{\sigma^{-1} \int d\Omega \frac{d\sigma}{d\Omega} \sin^2 \theta} \sim \# \text{ of collisions} \times \text{deflection weight}$$

→ $\sigma^{-1} \int d\Omega \frac{d\sigma}{d\Omega} \sin^2 \theta \equiv \sigma_{tr}/\sigma \rightarrow 2/3 \text{ for isotropic}$

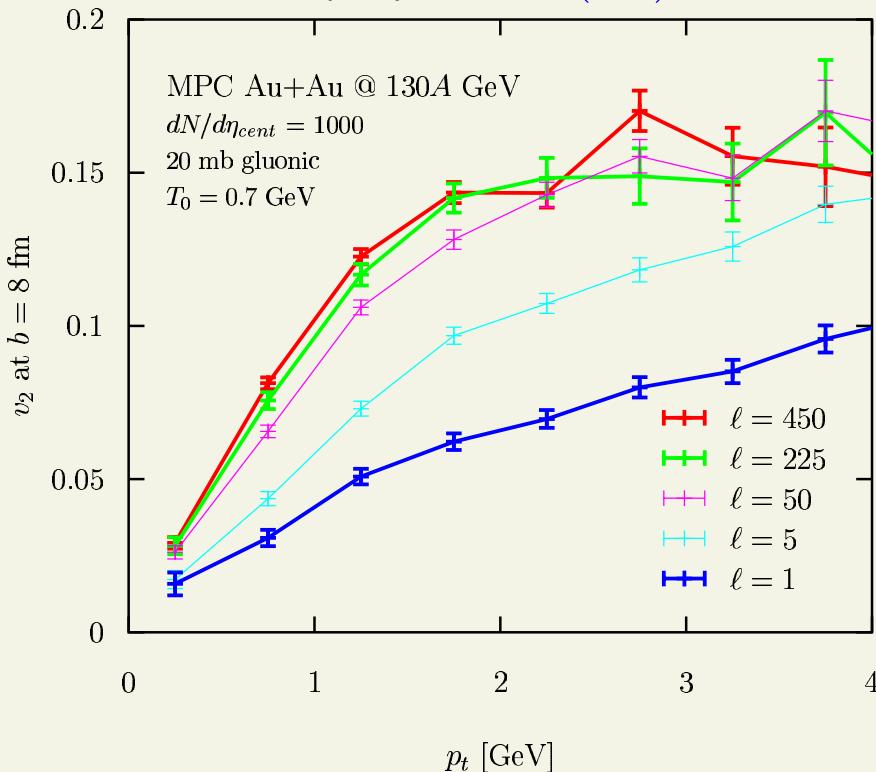
Nonlocal artifacts

Naive $2 \rightarrow 2$ cascade nonlocal - action at distance $d < \sqrt{\frac{\sigma}{\pi}}$

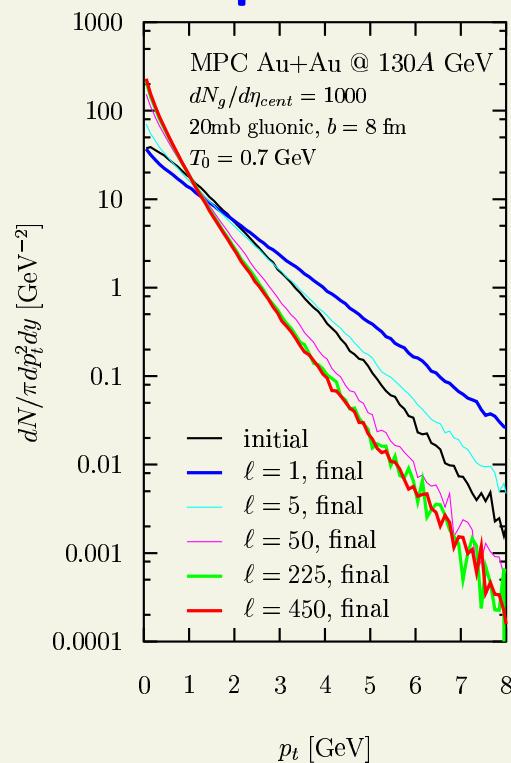


subdivision: rescale $f \rightarrow f \cdot \ell$, $\sigma \rightarrow \sigma/\ell$ $\Rightarrow d \propto \ell^{-1/2}$ local as $\ell \rightarrow \infty$

DM & Gyulassy ('02): $v_2(p_T)$



spectra



at RHIC: need subdivision $\ell \sim 200$ to eliminate large artifacts

→ computational challenge - CPU time scales as $\ell \sim 3/2$ per run → barely fits on PC

Connection to hydro

- **energy-momentum tensor:** $T^{\mu\nu}(x) = \sum_i \int \frac{d^3 p}{E} p^\mu p^\nu f_i(x, \vec{p})$

charge current: $N_c^\mu(x) = \sum_i \int \frac{d^3 p}{E} p^\mu c_i f_i(x, \vec{p})$

entropy current: $S^\mu(x) = \sum_i \int \frac{d^3 p}{E} p^\mu f_i(x, \vec{p}) \{ 1 - \ln[f_i(x, \vec{p}) h^3] \}$

conservation laws: $\partial_\mu T^{\mu\nu} = 0, \partial_\mu N_c^\mu = 0$

dissipation: $\partial_\mu S^\mu \geq 0 \Rightarrow \text{entropy production, in general}$

- **Ideal fluid approximation** $\lambda \rightarrow 0$: **local equilibrium** $f = (2\pi)^{-3} \exp[p_\mu u^\mu(x)/T(x)]$

$$T_{id}^{\mu\nu} = (e + p)u^\mu u^\nu - p g^{\mu\nu}$$

$$\partial_\mu S^\mu = 0 \Rightarrow \text{entropy conserved}$$

- **Navier-Stokes approx.** $\lambda \ll \text{length scales}$: **near equilibrium - slowly varying**

$$n_B = 0 \quad T_{NS}^{\mu\nu} = T_{id}^{\mu\nu} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha \rightarrow \text{shear & bulk viscosity}$$

$\eta \propto T/\sigma$

$$\partial_\mu S_{NS}^\mu = \frac{\eta}{2T}(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha)^2 + \frac{\zeta}{T}(\nabla_\mu u^\mu)^2 \rightarrow \text{entropy production}$$

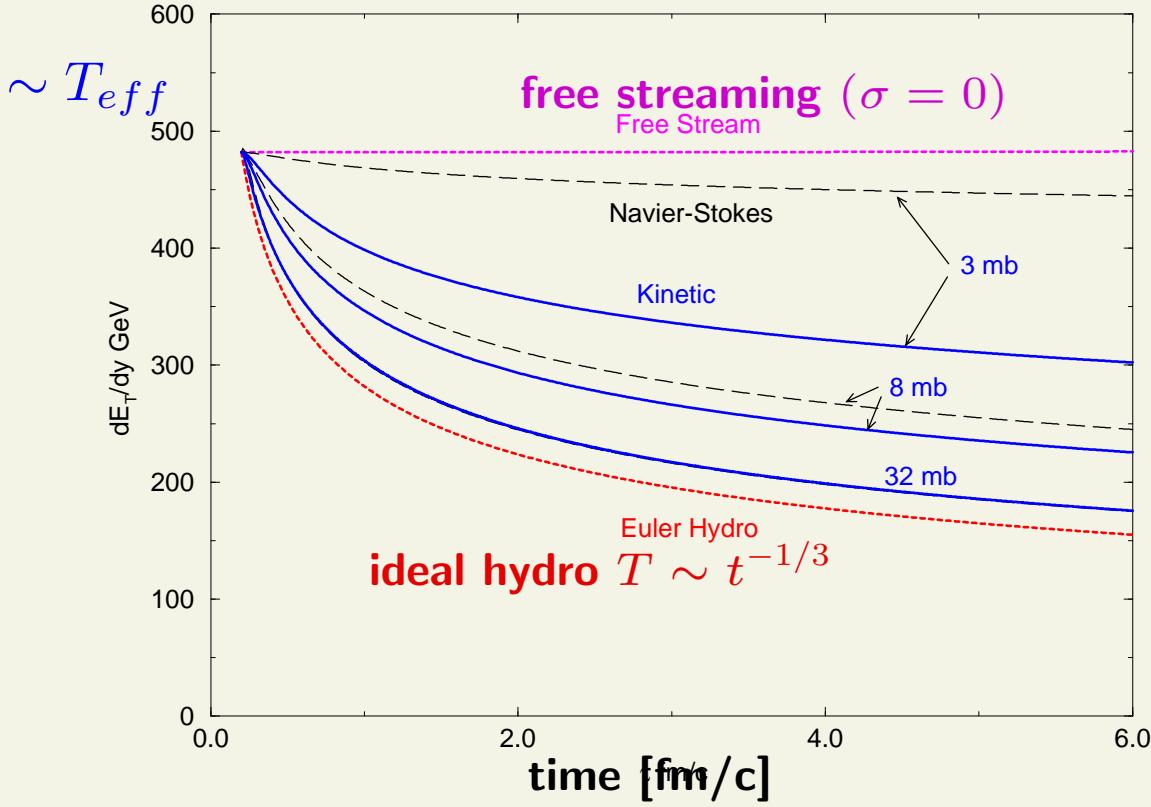
Collective signatures at RHIC

- cooling, steepening of spectra
- large elliptic flow
- heavy quark elliptic flow and chemistry
- baryon vs meson observables (B/M ratios, v_2 pattern)
- particle correlations - HBT

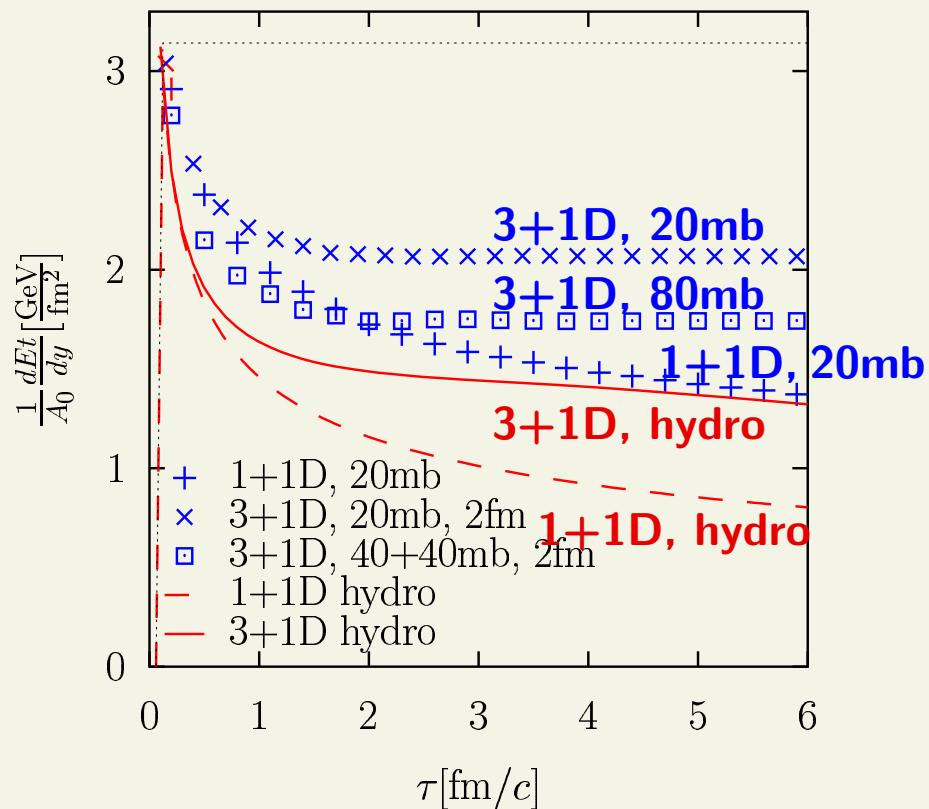
Cooling

Expanding systems cool due to $p dV$ work

Gyulassy, Pang & Zhang ('97): 1+1D



DM & Gyulassy ('00): 3+1D ($dN/d\eta = 210$)
MPC vs hydro (1+1D and 3+1D)

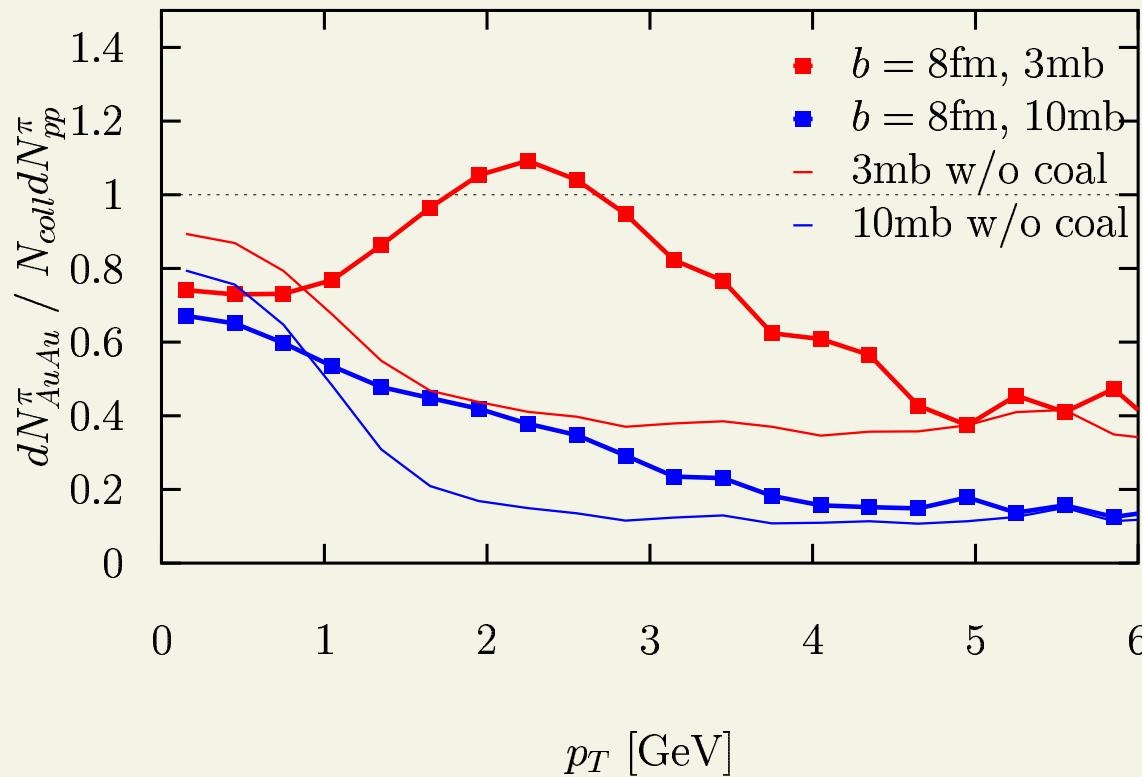


dissipation in transport slows cooling, especially in 3+1D

Opacity at RHIC

pion R_{AA} Au+Au @ 200 GeV, $b = 8$ fm (pQCD + saturation, $\tau_0 = 0.1$ fm/c, $dN^{cent}/d\eta = 2000$)

DM, JPG ('04):

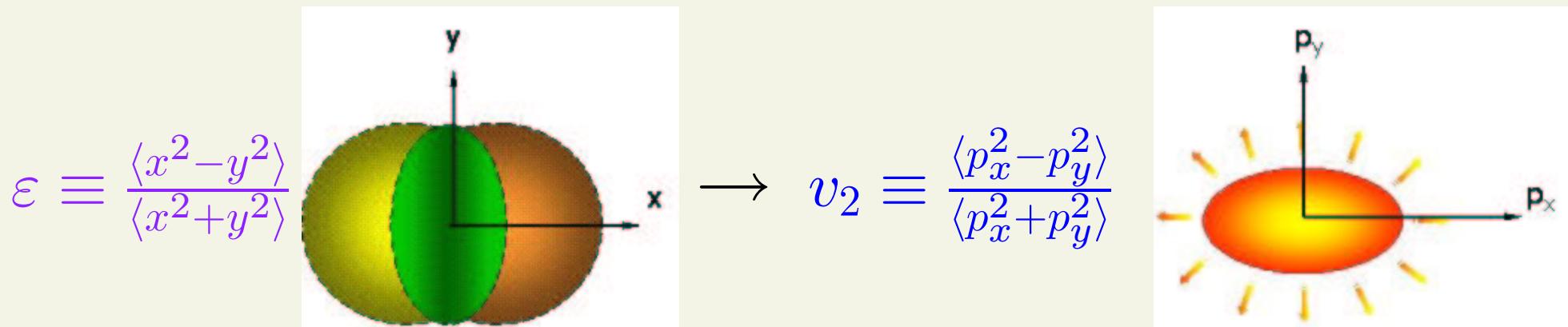


significant cooling, at high pT large elastic parton energy loss

opaque system - $\sigma \times dN^{cent}/d\eta \sim 6000 - 20000 \text{ mb}$ $\chi(b = 0) \sim 6 - 20$

Elliptic flow (v_2)

spatial anisotropy → final azimuthal momentum anisotropy

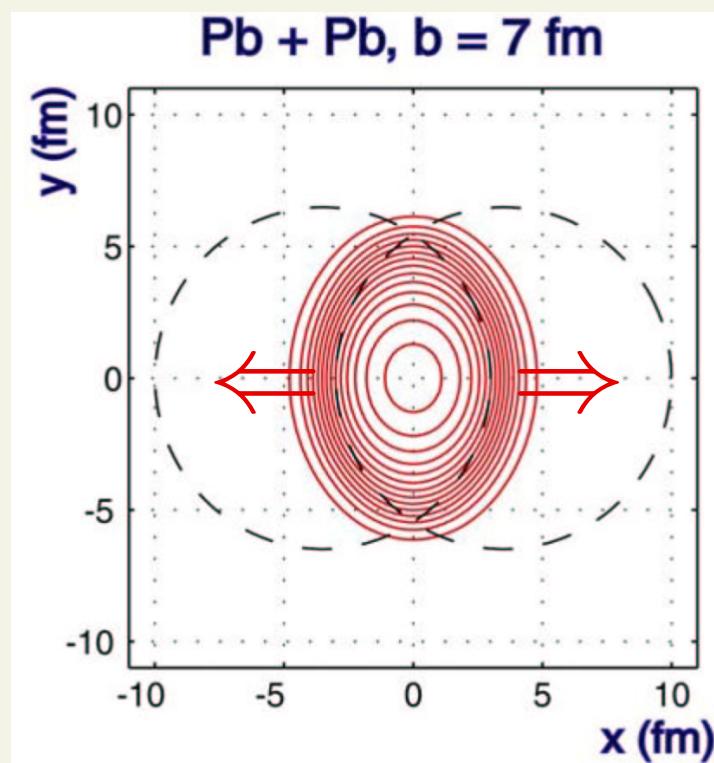


- measures strength of interactions
- self-quenching, develops at early times

What v_2 measures

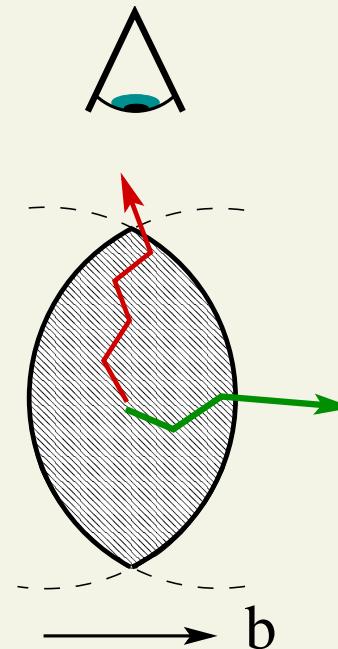
macroscopically: pressure gradients

$$\Delta \vec{F}/\Delta V = -\vec{\nabla}p$$

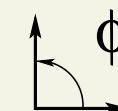


microscopically: transport opacity

smaller momenta
more deflection



beam axis view



larger
momenta
less
deflection

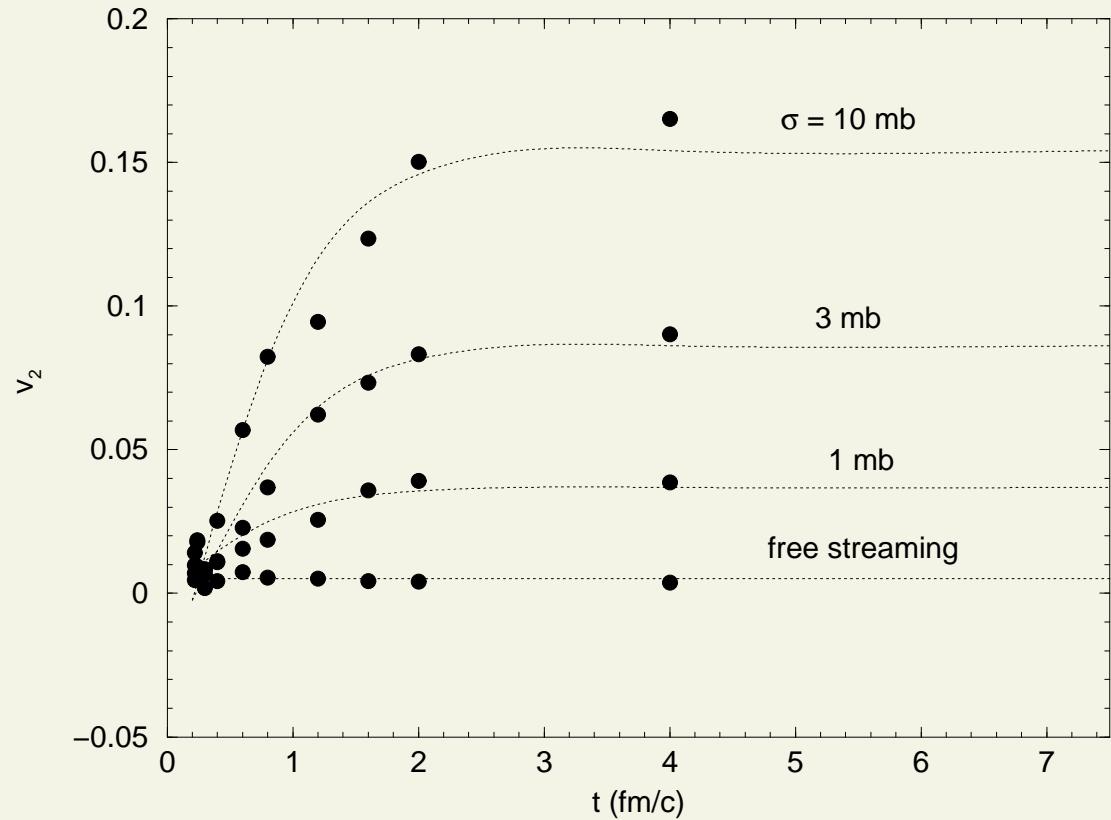


⇒ larger acceleration in impact parameter direction

variation in pathlength
⇒ momentum anisotropy v_2

v_2 builds up early

Zhang, Gyulassy & Ko ('99): **anisotropy builds up during first $\sim 2 \text{ fm}/c$**

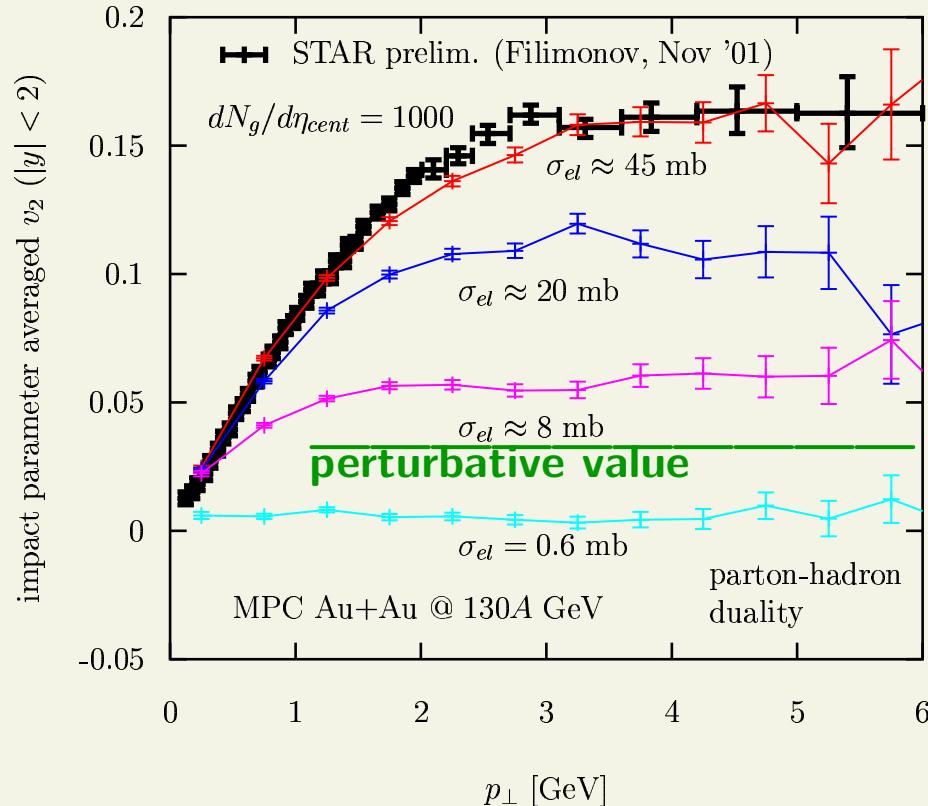


sharp cylinder $R = 5 \text{ fm}$, $\tau_0 = 0.2 \text{ fm}/c$, $b = 7.5 \text{ fm}$, $dN^{cent}/dy = 300$

Strong interactions at RHIC

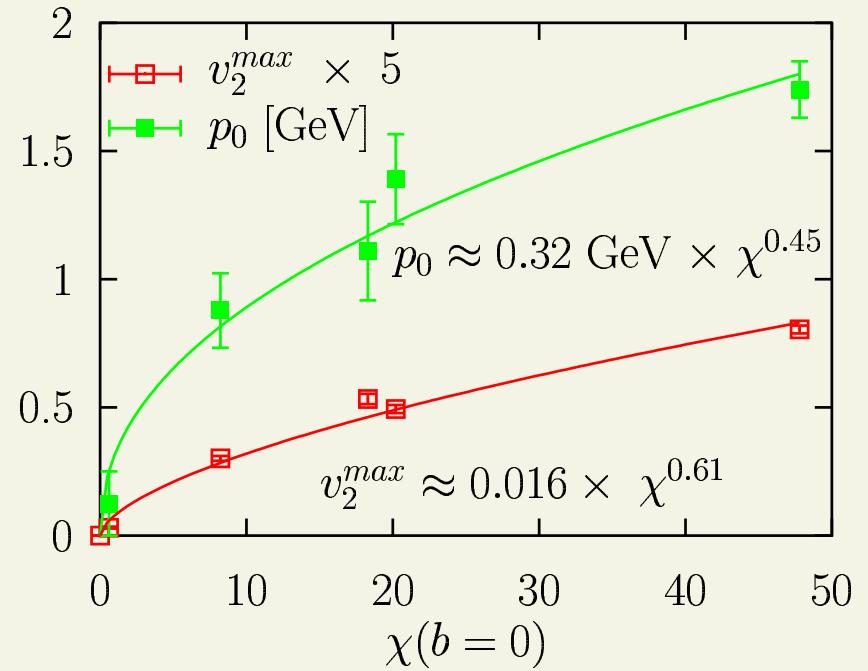
Au+Au @ 130 GeV, $b = 8$ fm

DM & Gyulassy, NPA 697 ('02): $v_2(p_T, \chi)$



nonlinear opacity dependence

$$v_2(p_T, \chi) \approx v_2^{max}(\chi) \tanh(p_T/p_0(\chi))$$

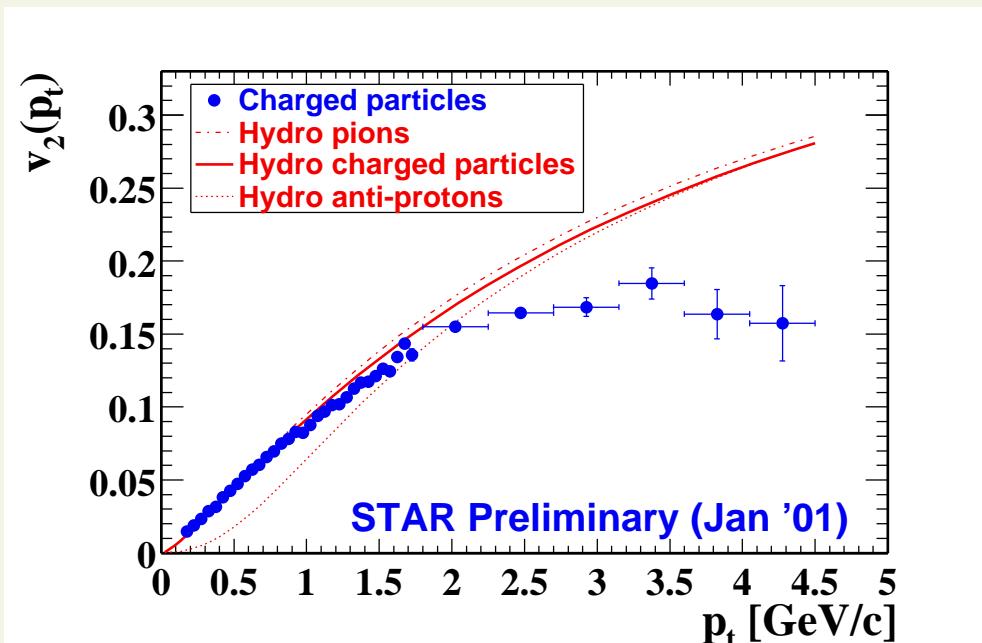


need $15 \times$ perturbative opacities - $\sigma_{el} \times dN_g/d\eta \approx 45$ mb $\times 1000 \Rightarrow$ sQGP

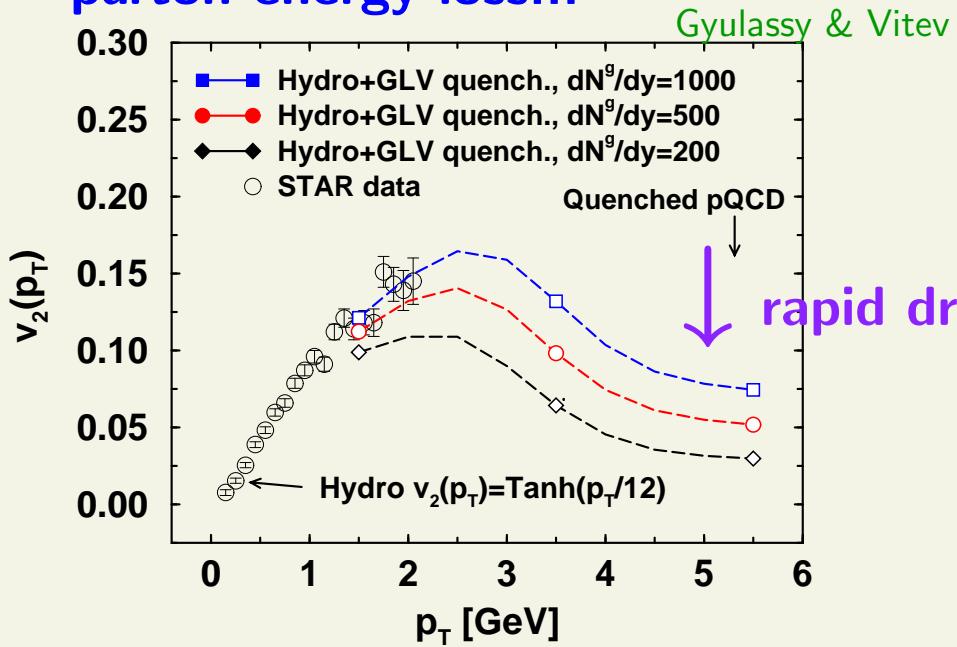
(saturated gluon $\frac{dN^{cent}}{d\eta} = 1000$, $T_{eff} \approx 0.7$ GeV, $\tau_0 = 0.1$ fm, 1 parton \rightarrow 1 π hadronization)

even larger opacity than from $R_{AA} \rightarrow$ PUZZLE #1 already DM '99 (OSCAR-II workshop)

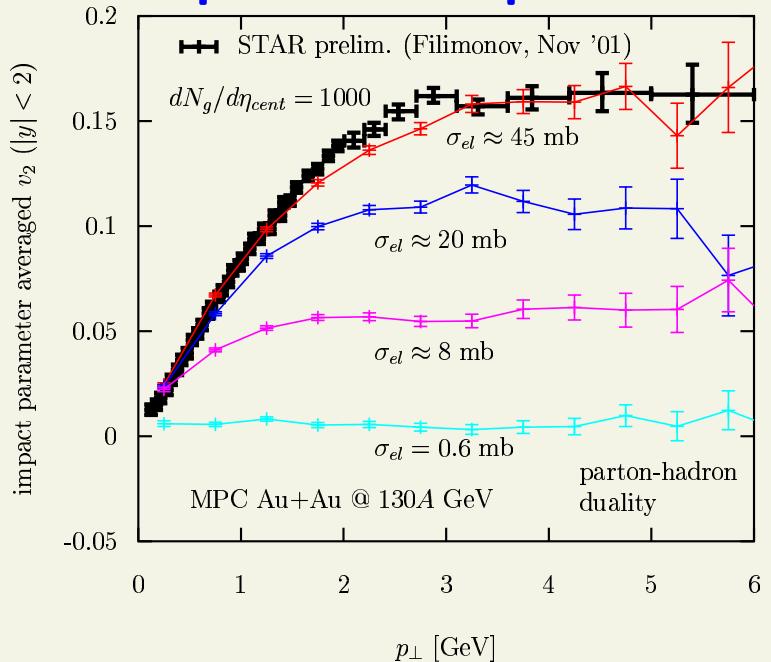
ideal hydrodynamics



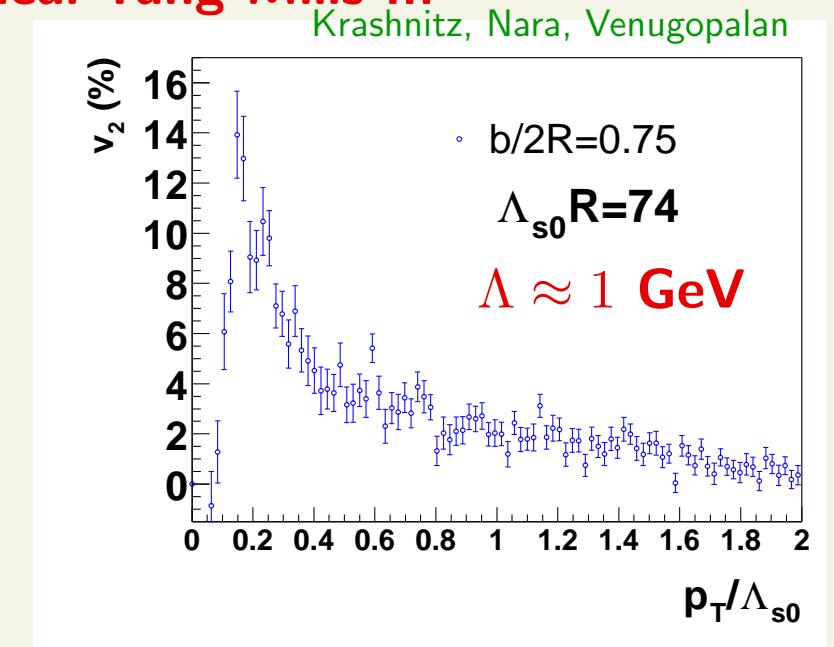
parton energy loss...



covariant parton transport

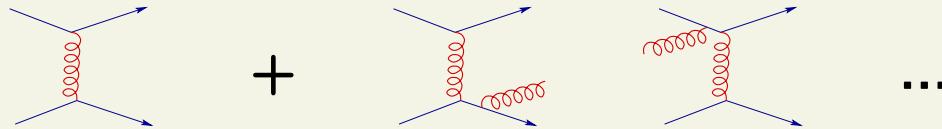


classical Yang-Mills ...

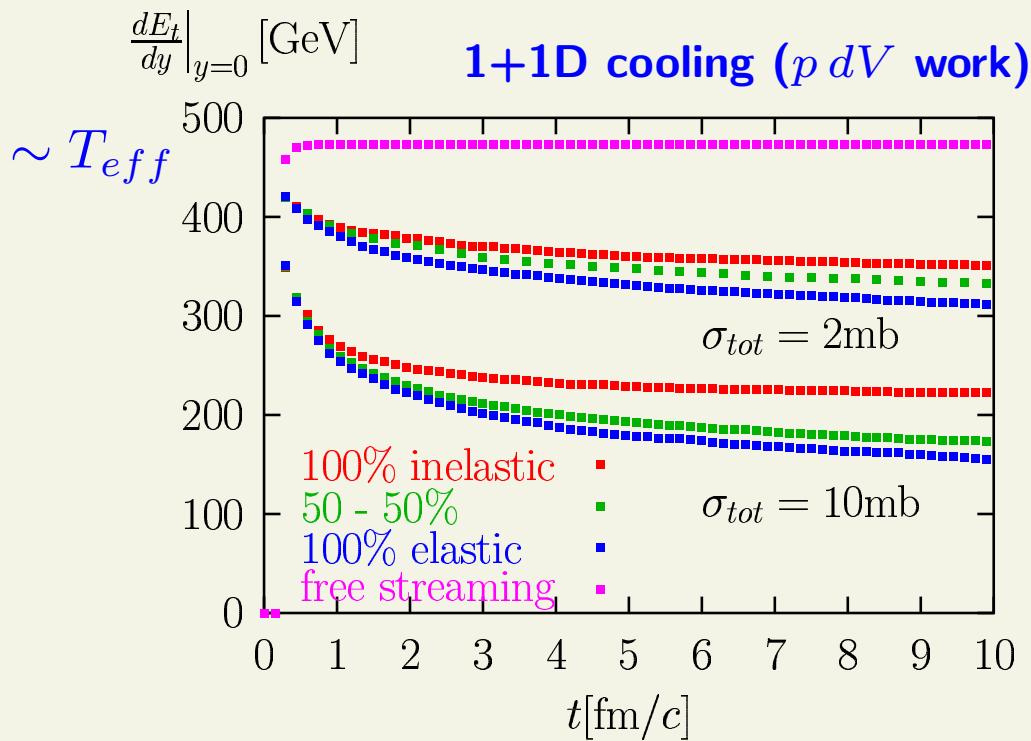


Radiative transport, $3 \leftrightarrow 2$

higher-order processes also contribute to thermalization



but enhance effective opacity only 2 – 3 times → still s-QGP



DM & Gyulassy, NPA 661, 236 ('99)

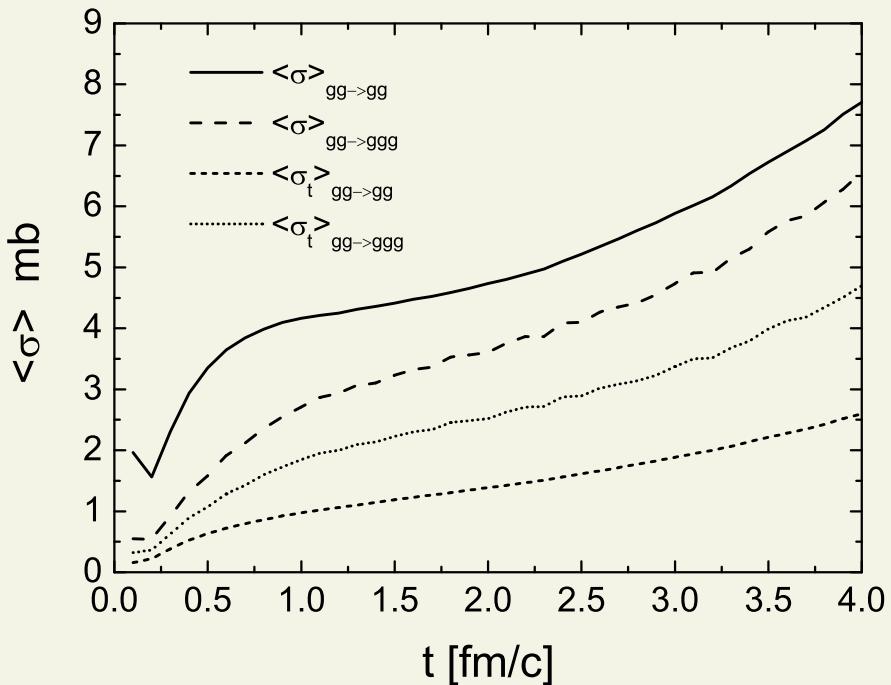
**fixed transport cross section
but vary degree of inelasticity**

100% elastic, 100% inelastic, 50-50%
 $2 \rightarrow 2$ $3 \leftrightarrow 2$ mixed

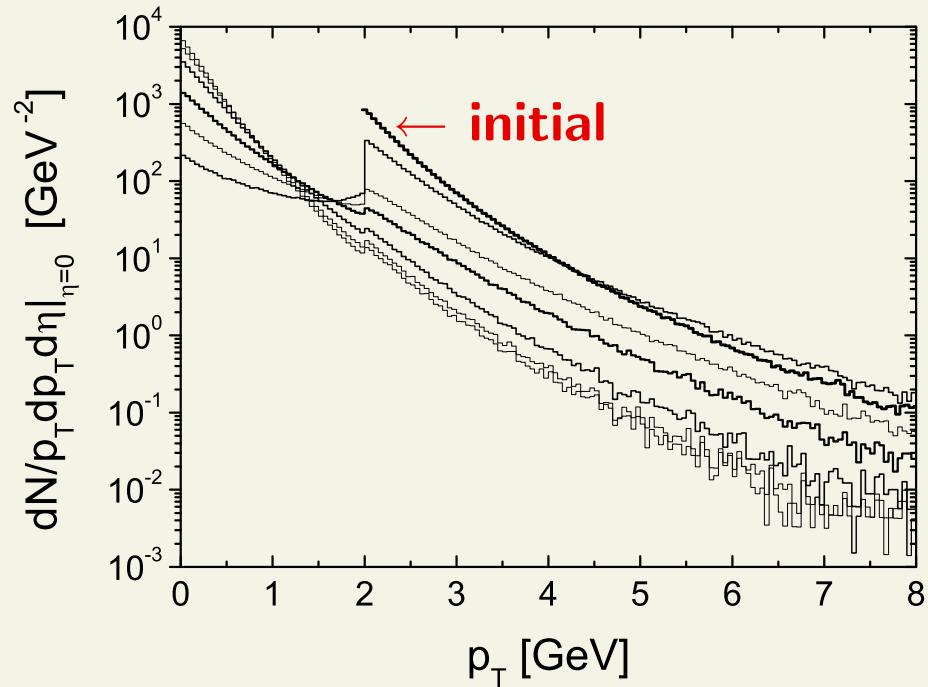
⇒ inelastic $3 \leftrightarrow 2$ is roughly same as elastic with same transport cross section

Greiner & Xu '04: claim thermalization time-scale $\tau \sim 2 - 3 \text{ fm}/c$

$2 \rightarrow 2, 2 \rightarrow 3$ transport cross sections



spectra vs. time



inel. roughly doubles $\sigma_{tr} \leftarrow \text{OK}$

rapid cooling via $2 \rightarrow 3$, because assumed there is nobody below 2 GeV(!)

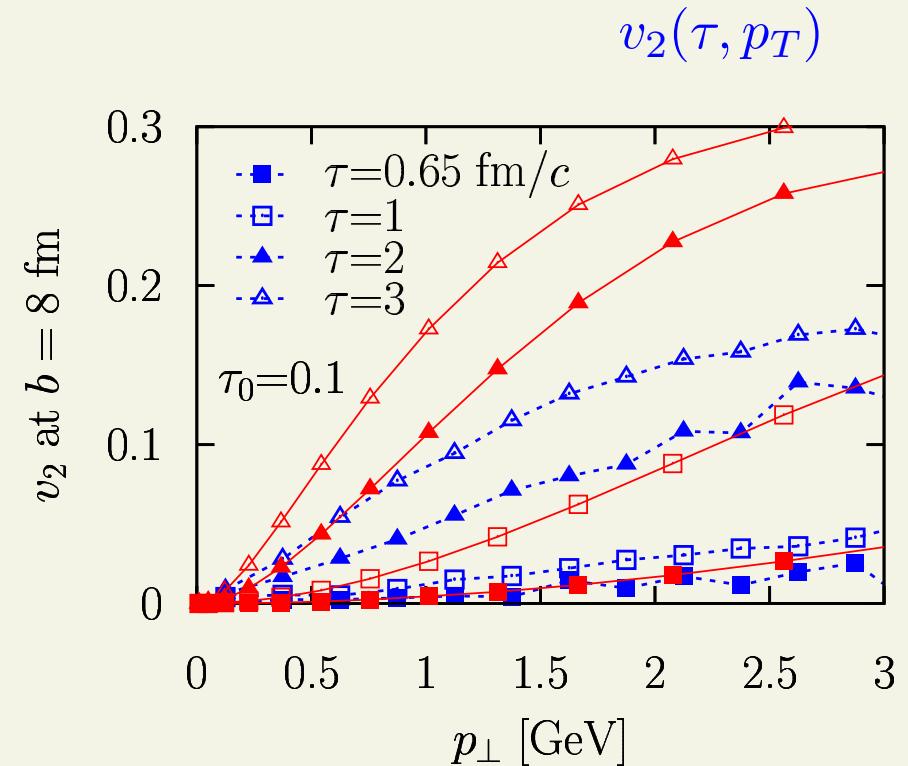
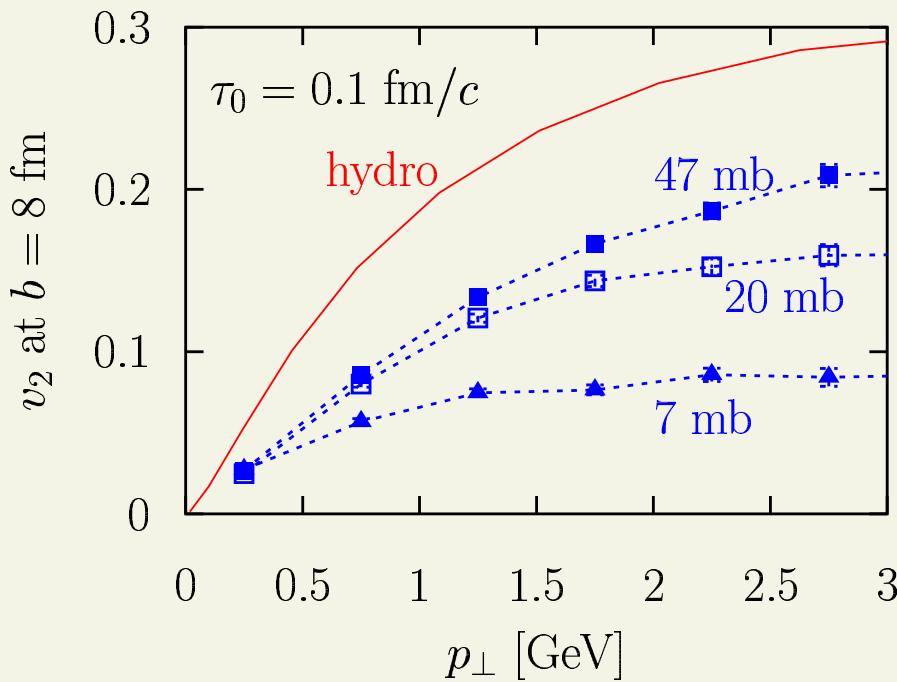
\Rightarrow driven mostly by phasespace, not collective “pressure” \Rightarrow little v_2

in contrast DM & Gyulassy: low- p_T region initially filled

No, still not ideal fluid(!)

dissipation reduces v_2 by 30 – 50% even for $\sigma_{gg \rightarrow gg} \sim 50$ mb

DM & Huovinen, PRL94 ('05): final $v_2(p_T)$



→ dense, strongly-interacting system, but still dissipative

Almost perfect fluid

$\sigma_{gg} \sim 50 \text{ mb} \leftrightarrow \lambda_{MFP} \sim 0.08 \text{ fm}$ - is likely the best one can get

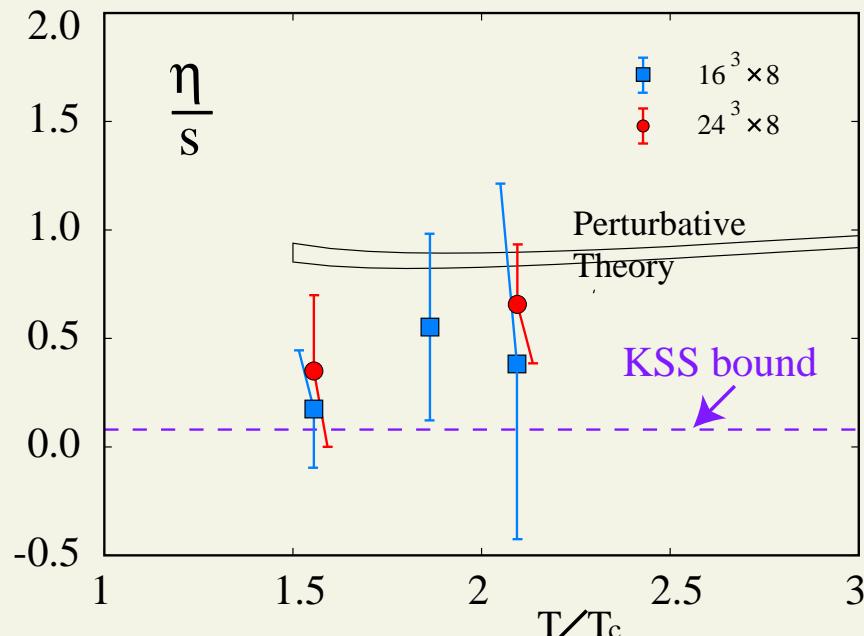
quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

\Rightarrow kinetic theory: $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz '85

viscosity: $\eta = s \frac{\lambda T}{5} \Rightarrow$ minimal viscosity $\eta/s \geq 1/15$

parton transport + large RHIC v_2 indicate QGP is most ideal fluid possible

Nakamura & Sakai ('04): viscosity from lattice QCD

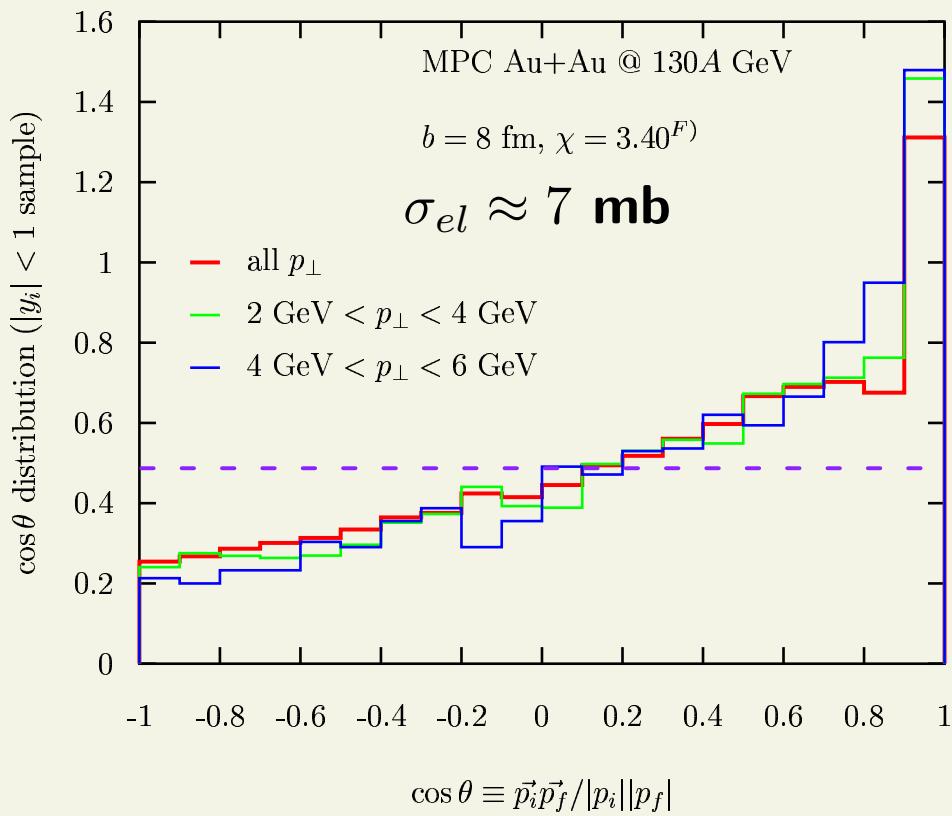


VERY difficult calculation - will take long to converge with numerics

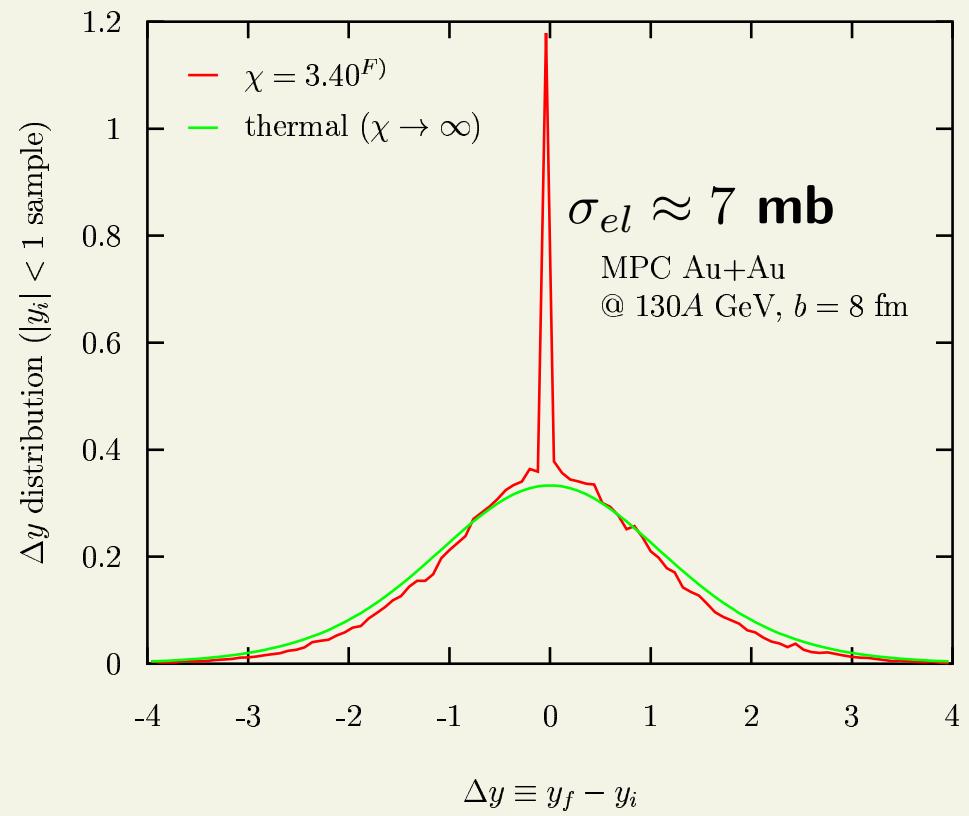
← string theory conjecture: $\eta/s \geq 1/4\pi$
 Son et al ('02), ('04) - proof for $\mathcal{N} = 4$ SYM
 - RHIC can test this theory bound

Significant randomization

a) deflection angle $\vec{p}_i \angle \vec{p}_f$



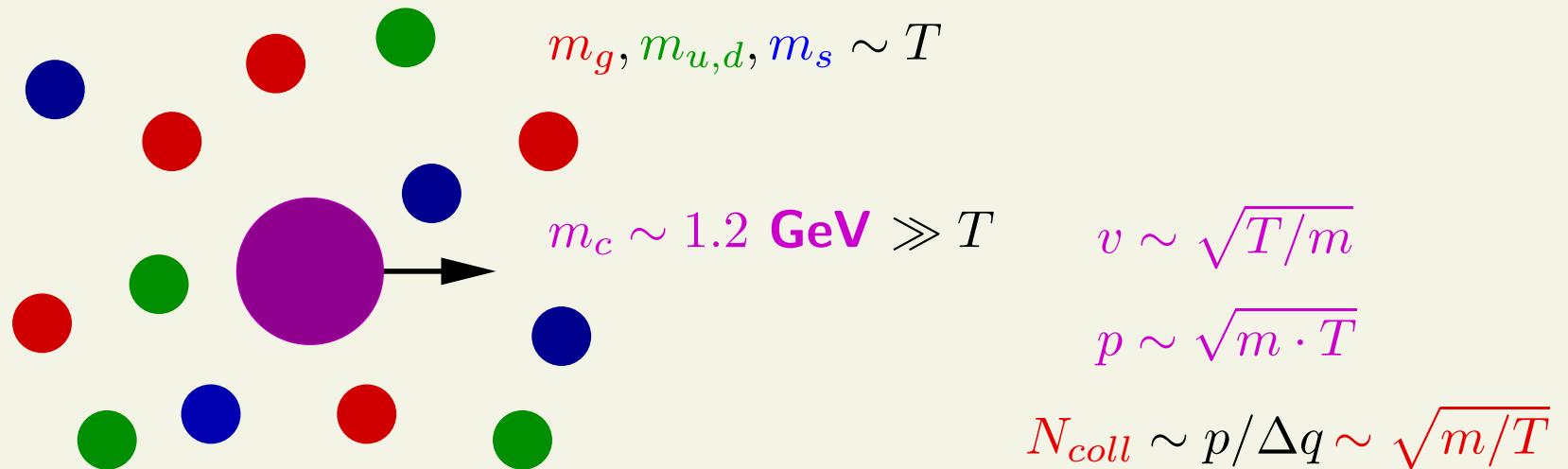
b) rapidity shift $y_f - y_i$



light parton momenta randomize to large degree, already for $\sigma \sim 7 \text{ mb}$ ($\chi \sim 7$)

Cross-check: heavy quarks

~ “Brownian motion” in plasma



charm quarks **very heavy** \Rightarrow need more collisions to randomize

\Rightarrow at low momenta: expect reduced anisotropy v_2

\Rightarrow at high momenta: mass difference should not matter as $m/p \rightarrow 0$

assume: all $2 \rightarrow 2$ processes are enhanced by same factor in opaque plasma

based on Combridge NPB 151 ('79) 429:

$$\begin{aligned}\sigma_{gg \rightarrow q\bar{q}} &= \frac{2r}{27} \frac{1+r}{1+2r} \ln\left(1 + \frac{1}{r}\right) \sigma_{gg \rightarrow gg}, \quad \sigma_{q_i\bar{q}_i \rightarrow q_j\bar{q}_j} = \frac{16r}{243} \sigma_{gg \rightarrow gg} \\ \sigma_{gg \rightarrow c\bar{c}} &= \frac{2r}{27} \Theta(1-4R) \left[(1+4R+R^2) \ln \frac{1+\sqrt{1-4R}}{1-\sqrt{1-4R}} - (7+3R) \frac{\sqrt{1-4R}}{4} \right] \sigma_{gg \rightarrow gg} \\ \sigma_{q\bar{q} \rightarrow c\bar{c}} &= \frac{16r}{243} \Theta(1-4R)(1+2R)\sqrt{1-4R} \sigma_{gg \rightarrow gg}\end{aligned}$$

where $r \equiv \mu_D^2/s$, $R \equiv M_c^2/s$

take $\mu_D = 0.7 \text{ GeV}$, $M_c = 1.2 \text{ GeV}$

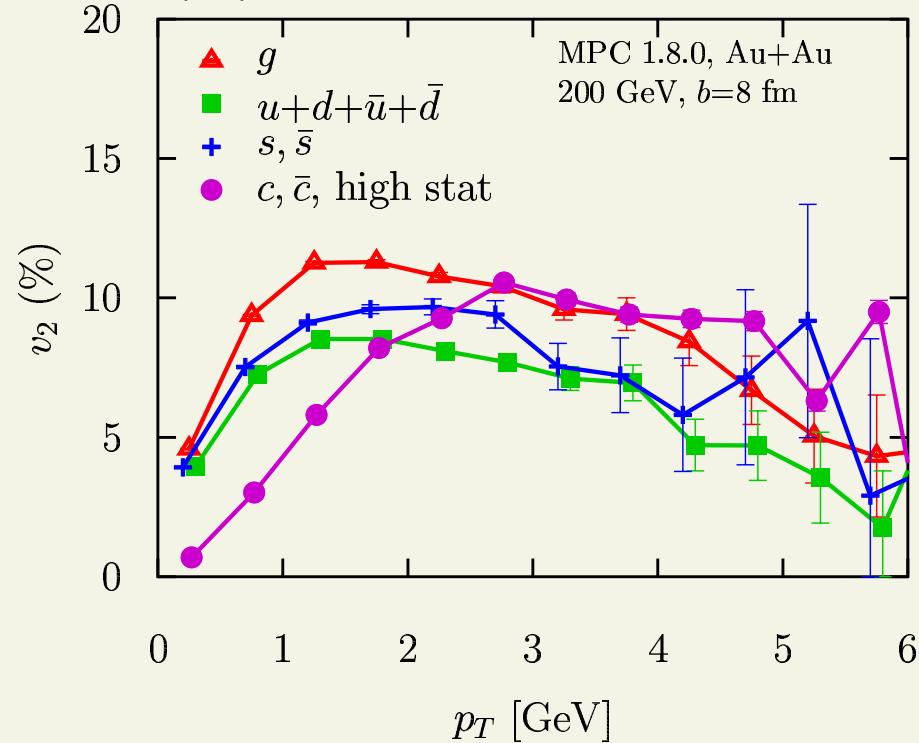
Predicted charm flow(!)

parton transport MPC 1.8.0

vs

indirect $D(qc)$ meson measurement:

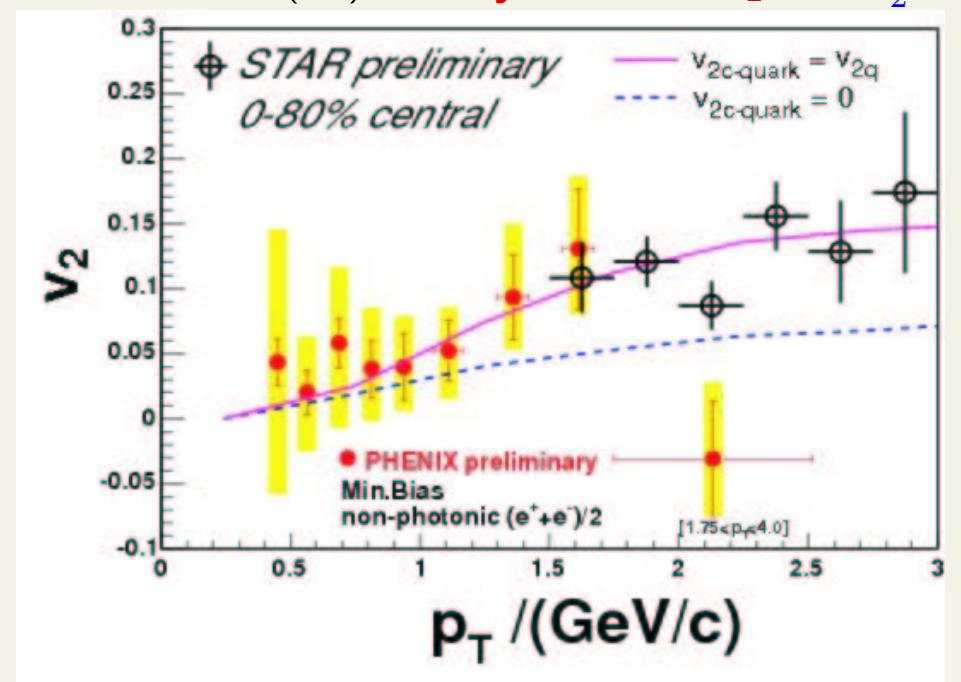
DM, JPG ('04): parton v_2



elastic & inel. $2 \rightarrow 2$

$6 \times$ perturbative opacities

PHENIX, STAR ('04): decay electron $v_2 \approx v_2^D$



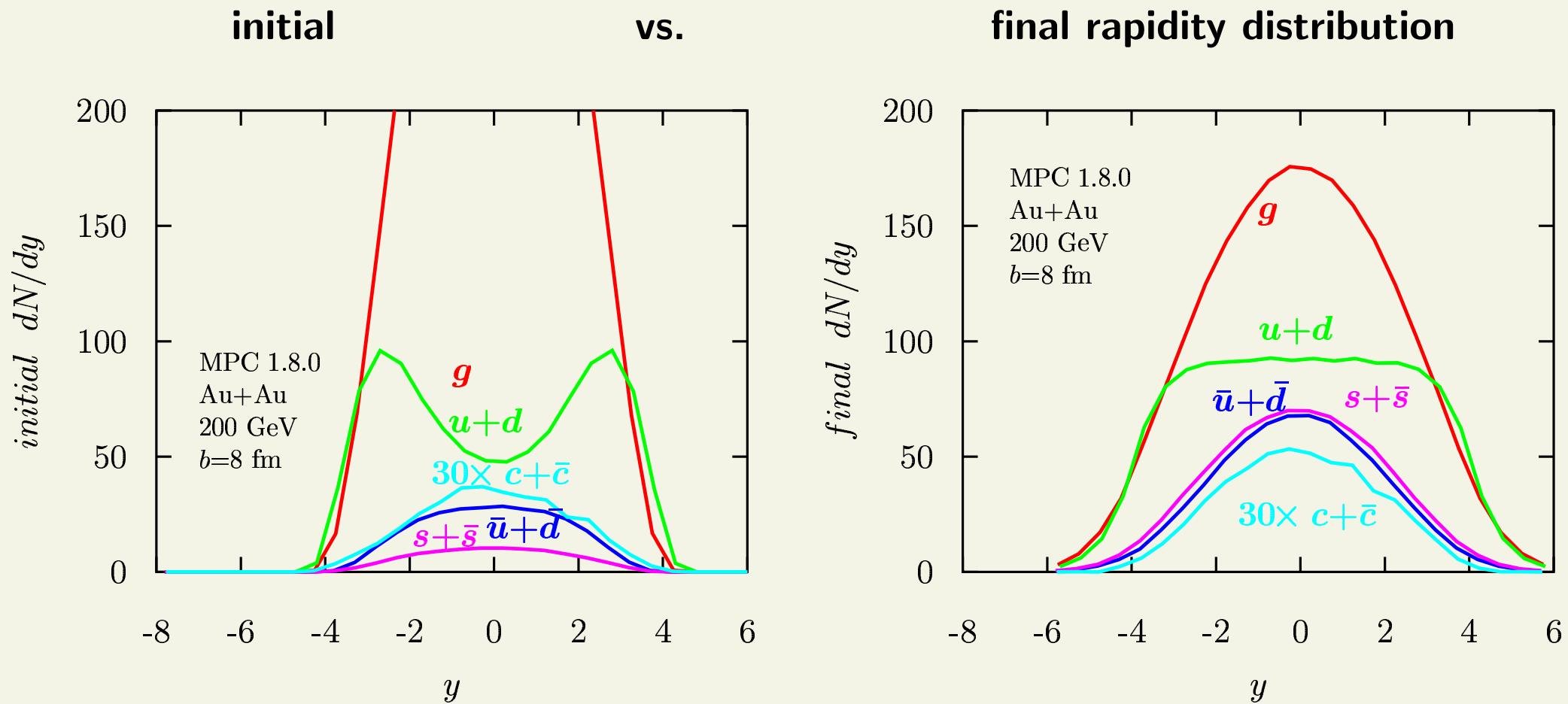
uses decay electrons: $D \rightarrow K^{(*)} \nu e$

e's from hadron decays and γ -conversion subtracted
 \equiv "non-photonic"

qualitative agreement with data, detailed studies will follow - $v_2(b, \chi, \sqrt{s})$

also expect secondary charm production from opaque plasma

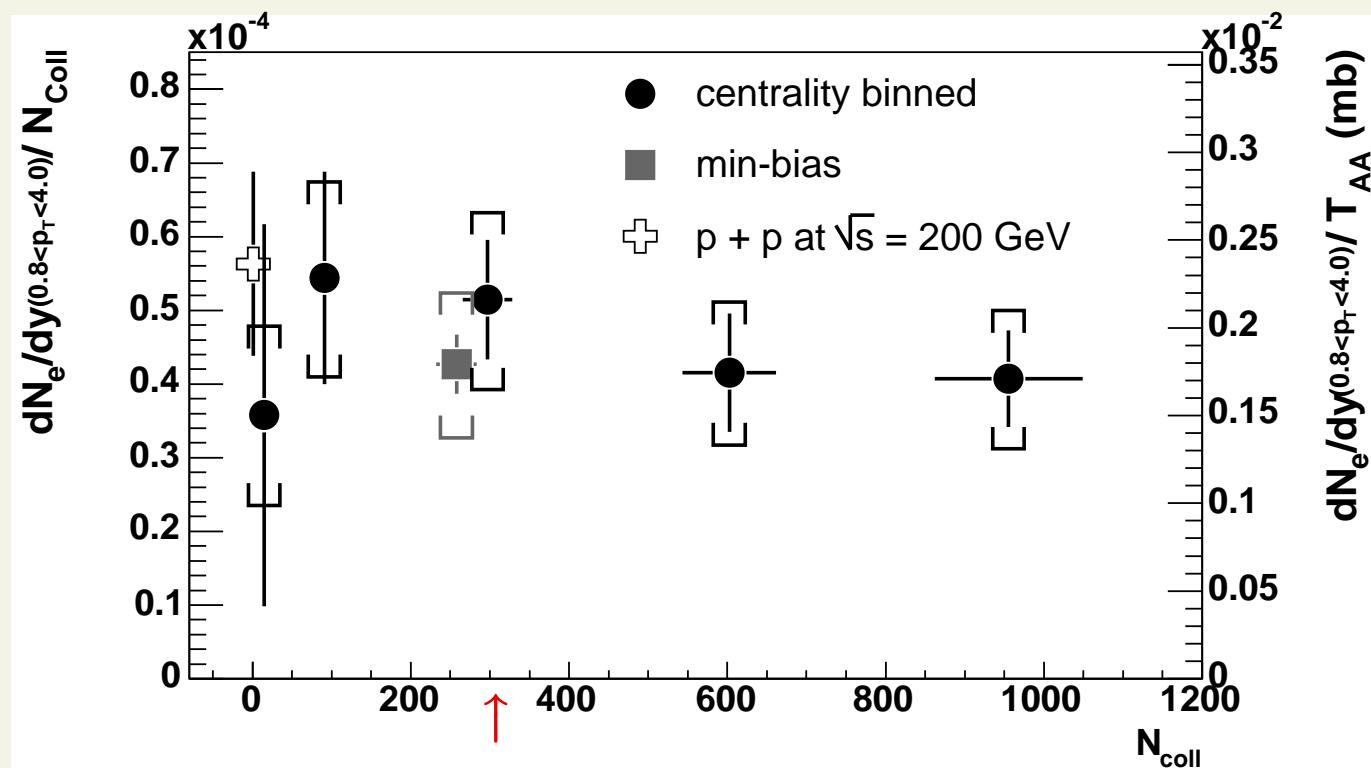
DM, JPG ('04):



- roughly half the glue fuse to $q\bar{q}$
- extra 40 – 50% charm yield due to secondary production
- strangeness is up by much more, factor 5 or so

PUZZLE #2 - data indicate no secondary charm (N_{coll} scaled p+p)

PHENIX, PRL 94 (2005) 082301



Soft physics tails at high pT(!)

partons can end up with some final parton momentum (p_T , y) in three ways:

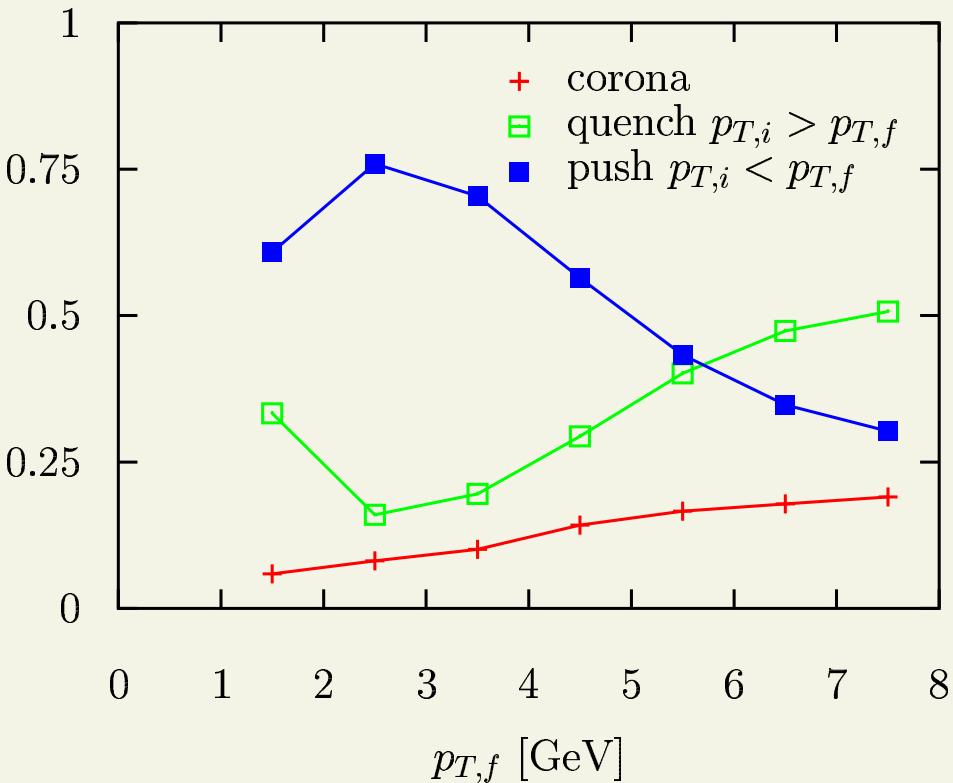
- escape with no interaction - corona
- interact and lose energy - quench
- 3rd possibility: interact and gain energy - “push”

in opaque plasma, gain component can be relevant at surprisingly high p_T , pushing “pure” hard physics out to $p_T \gtrsim 10$ GeV

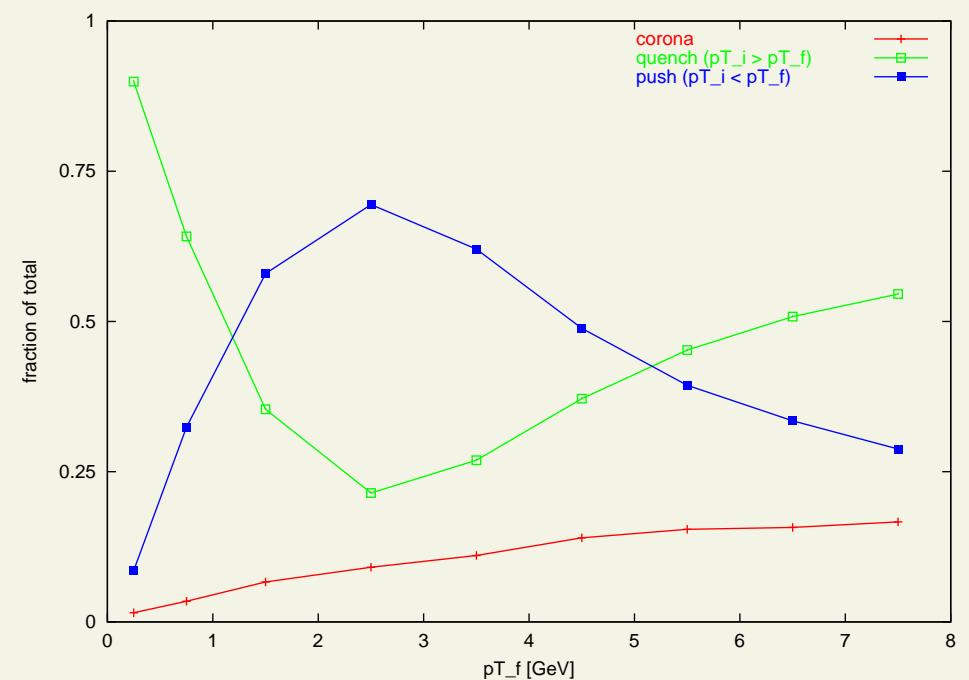
study using MPC 1.8.0 w/ elastic and inelastic $2 \rightarrow 2$, $dN^{cent}/d\eta = 2000$

fractions from corona, quench, push vs pT, ($|y_f| < 1$)

DM, nucl-th/0503051: $\sigma_{gg} = 10 \text{ mb}$



$\sigma_{gg} = 5 \text{ mb}$



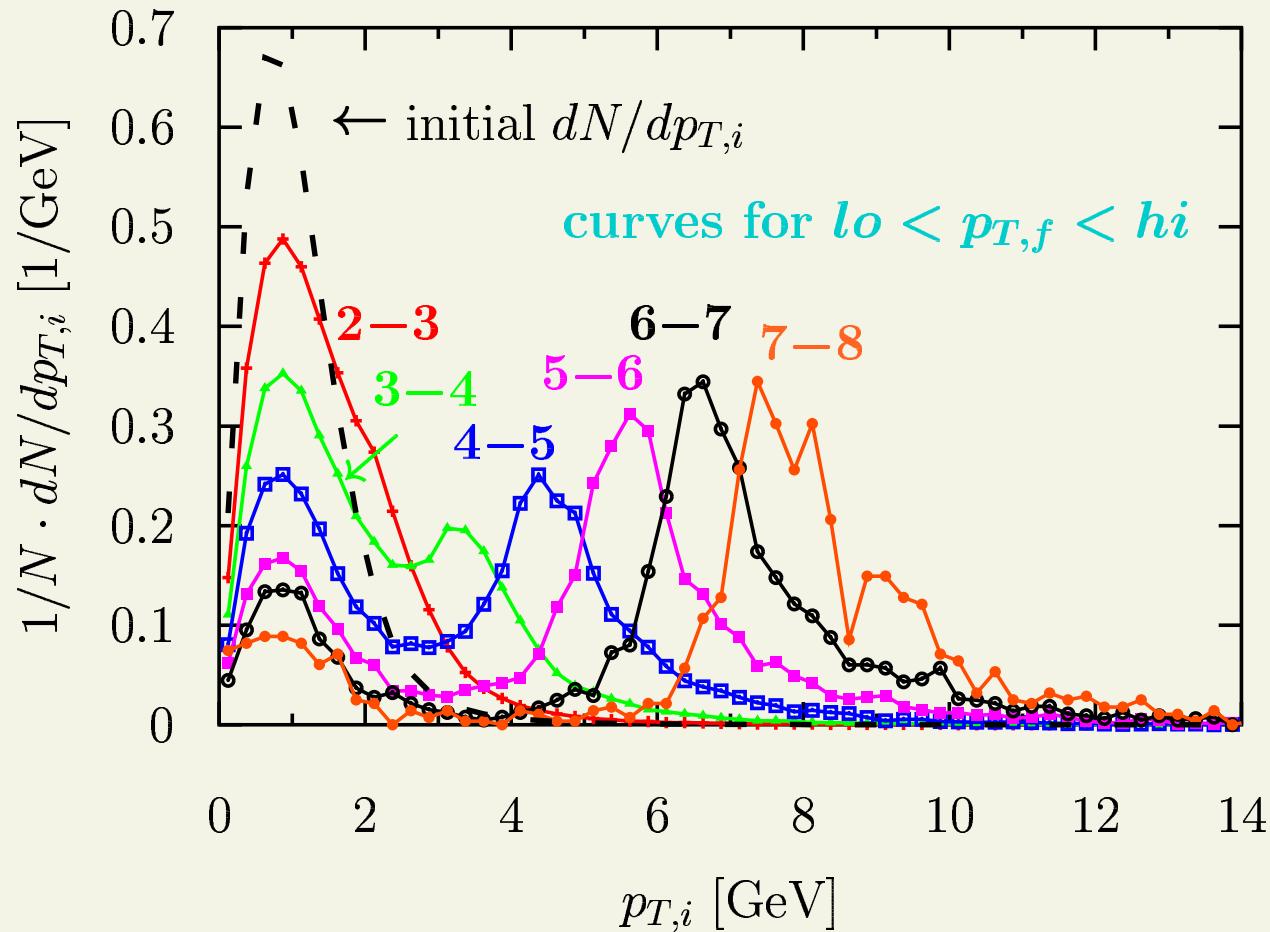
corona and “push” are significant even at $p_{T,parton} \sim 8 \text{ GeV}$

fractions show surprisingly weak opacity dependence

distribution of initial momenta for fixed final momentum bins, $|y_{fin}| < 1$

(only quench + “push” plotted, normalized)

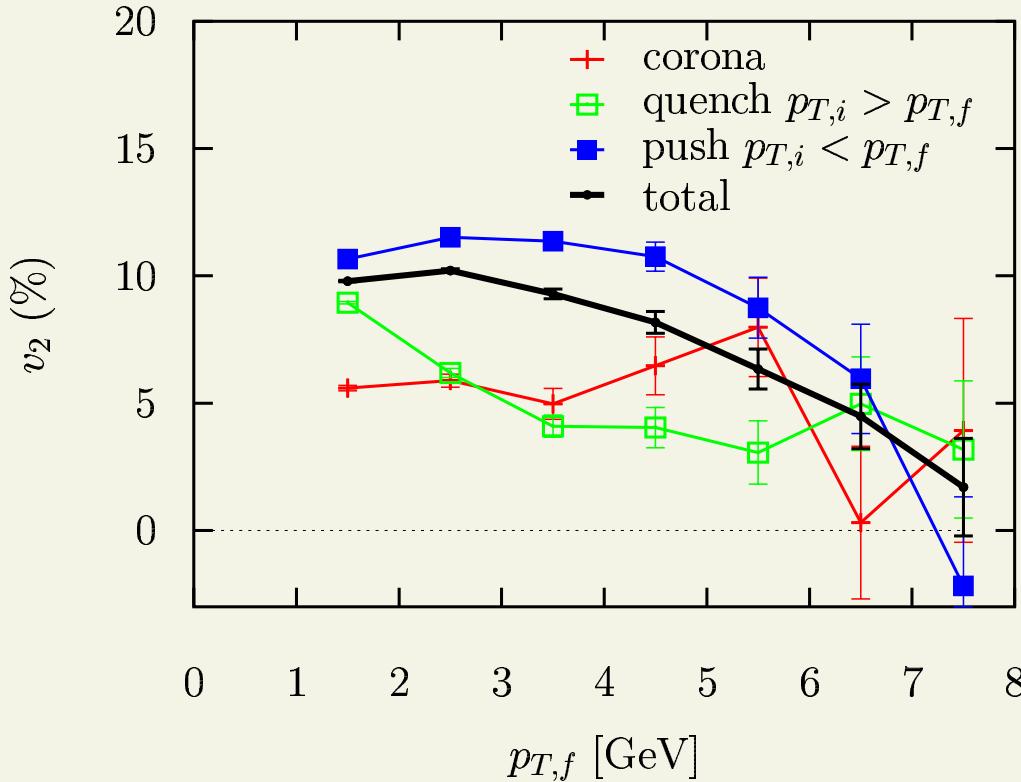
DM, nucl-th/0503051: $\sigma_{gg} = 10 \text{ mb}$



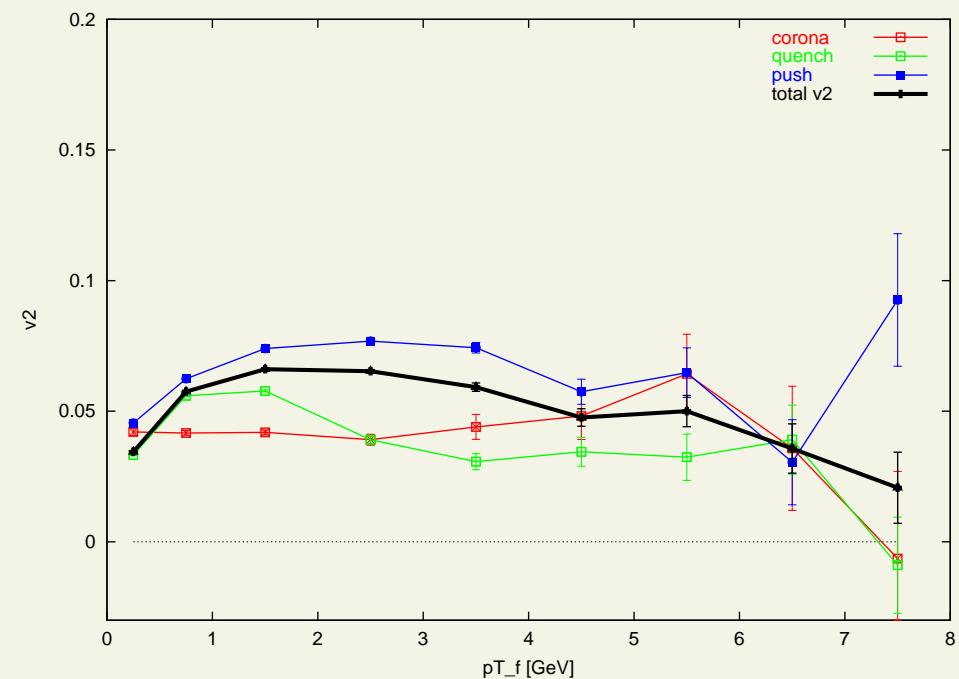
“lucky” $p_{T,i} \sim 1$ GeV soft partons can end up at $p_T \sim 5 - 6 - 7$ GeV

elliptic flow contributions vs pT

DM, nucl-th/0503051: $\sigma_{gg} = 10 \text{ mb}$



$\sigma_{gg} = 5 \text{ mb}$



rapid v_2 drop from quench at high p_T is compensated by large v_2 of “pushed-up” partons

combined $v_2(p_T)$ decreases more slowly at high p_T and can exceed “geometric” (extreme absorption) bounds Shuryak ('04), Voloshin ('04)

Where do the high opacities come from?

BIGGEST PUZZLE

- **strong correlations?** - critical scattering, **(quasi)bound states** Shuryak, Zahed et al ('04)
- **plasma instabilities?** - quark-gluon **E & M** Mrowczynski '93, Arnold et al ('04)
- **only apparent (hadronization effects)?** - coalescence ←

Quark coalescence

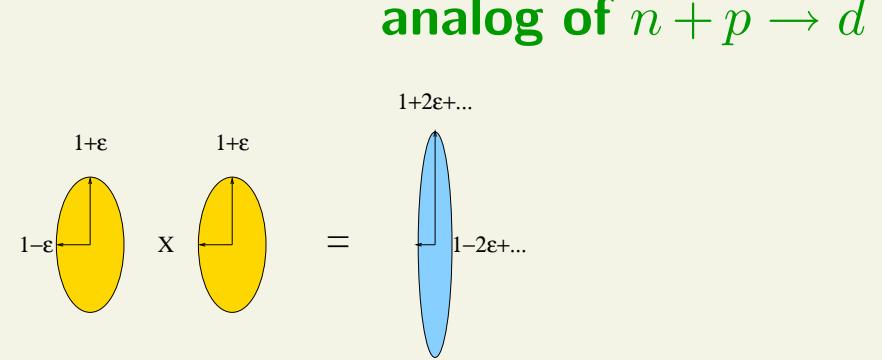
Ko, Lin, Voloshin, DM, Greco, Levai, Mueller, Fries, Bass, Nonaka, Asakawa ...

coalescence of comoving quarks: $q\bar{q} \rightleftharpoons M$ $3q \rightleftharpoons B$

DM & Voloshin, PRL91 ('03)

$$\frac{dN_M(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/2)}{d\phi} \right]^2$$

$$\frac{dN_B(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/3)}{d\phi} \right]^3$$

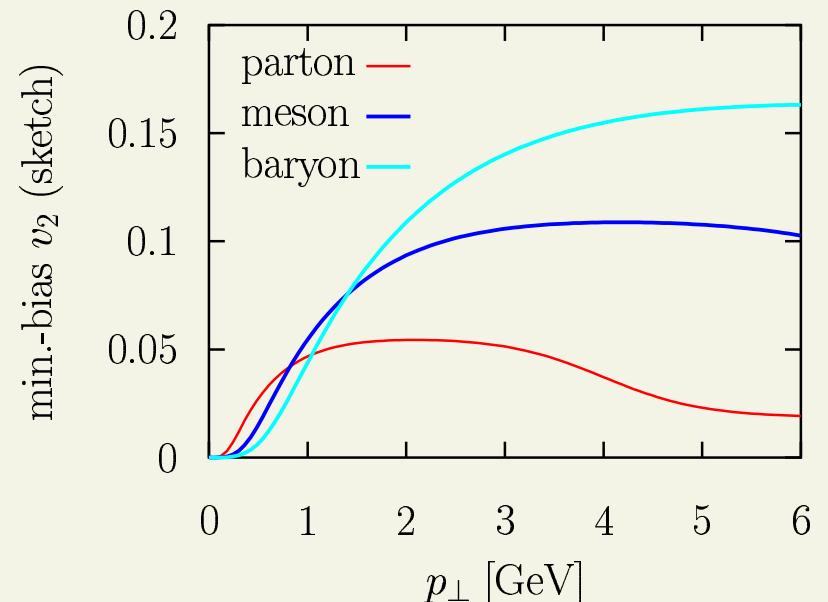


squared/cubed probability → amplified v_2

$$v_2^{\text{hadron}}(p_\perp) \approx n \times v_2^{\text{quark}}(p_\perp/n)$$

$3 \times$ for baryons } 50% larger v_2
 $2 \times$ for mesons } for baryons

→ 5× for pentaquark, 6× for deuteron



amplification greatly reduces opacities needed to reproduce v_2 data

coalescence can help R_{AA} vs v_2 puzzle due to reversal in trends

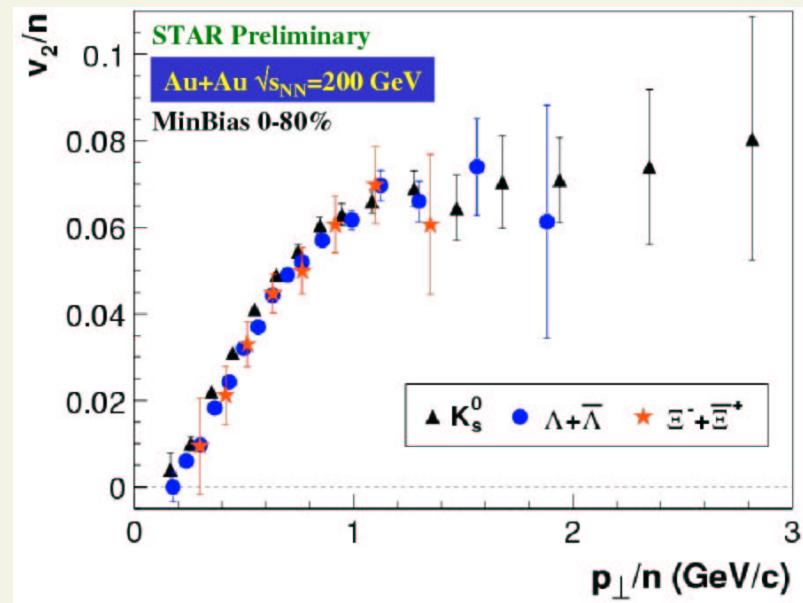
effect of opacity increase: v_2 up, R_{AA} down

effect of coalescence: v_2 up, R_{AA} up at hadronization

picture hangs together nicely, if quark final state is a fit parameter

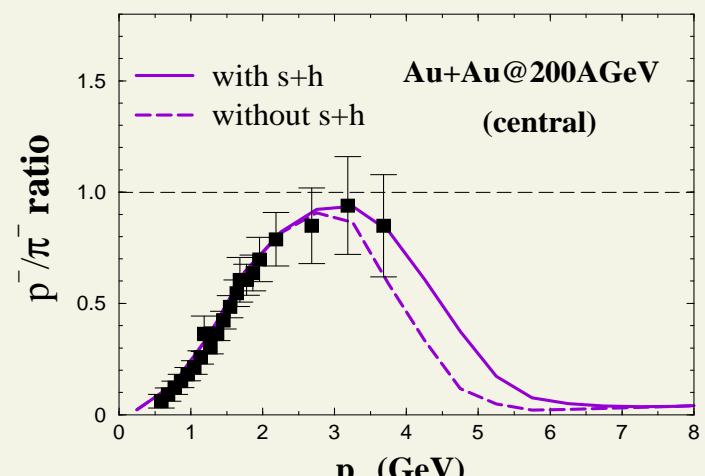
quark number scaling at RHIC

Castillo [STAR], HIC03: K_S^0, Λ, Ξ



pion/proton ratio

Greco, Ko, Levai, PRL90 ('03):



coalescence formula

$$\frac{dN_M(\vec{p})}{d^3p} = g_M \int (\prod_{i=1,2} d^3x_i d^3p_i) W_M(x_1-x_2, \vec{p}_1 - \vec{p}_2) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

$$\frac{dN_B(\vec{p})}{d^3p} = g_B \int (\prod_{i=1,2,3} d^3x_i d^3p_i) W_B(x_{12}, x_{13}, \vec{p}_{12}, \vec{p}_{13}) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) f_\gamma(\vec{p}_3, x_3) \delta^3(\vec{p} - \sum \vec{p}_i)$$

hadron yield space-time hadron wave-fn. quark distributions

gives v_2 scaling trivially if:

1. no other hadronization channels play a role
2. narrow wave functions $W \sim \delta^3(\Delta x) \delta^3(\Delta p)$
3. only small local harmonic modulations $|v_2(\mathbf{x})| \ll 1, |v_n(\mathbf{x})| \ll 1$

$$v_2^{Meson}(p_T) = \frac{2 \langle f_q^2(\mathbf{x}, p_T/2) v_{2,q}(\mathbf{x}, p_T) \rangle_{\mathbf{x}}}{\langle f_q^2(\mathbf{x}, p_T/2) \rangle_{\mathbf{x}}}$$

$$v_2^{Baryon}(p_T) = \frac{3 \langle f_q^3(\mathbf{x}, p_T/3) v_{2,q}(\mathbf{x}, p_T) \rangle_{\mathbf{x}}}{\langle f_q^3(\mathbf{x}, p_T/3) \rangle_{\mathbf{x}}}$$

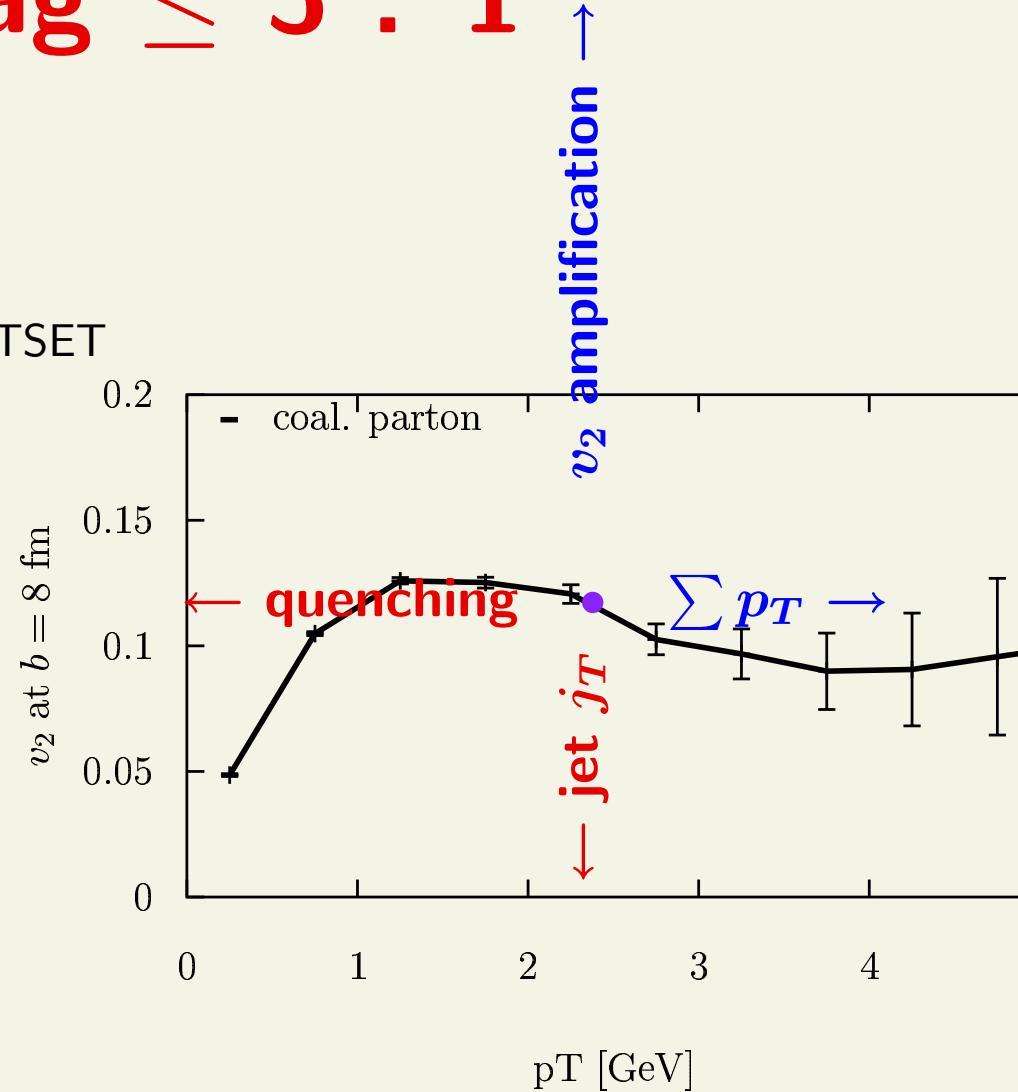
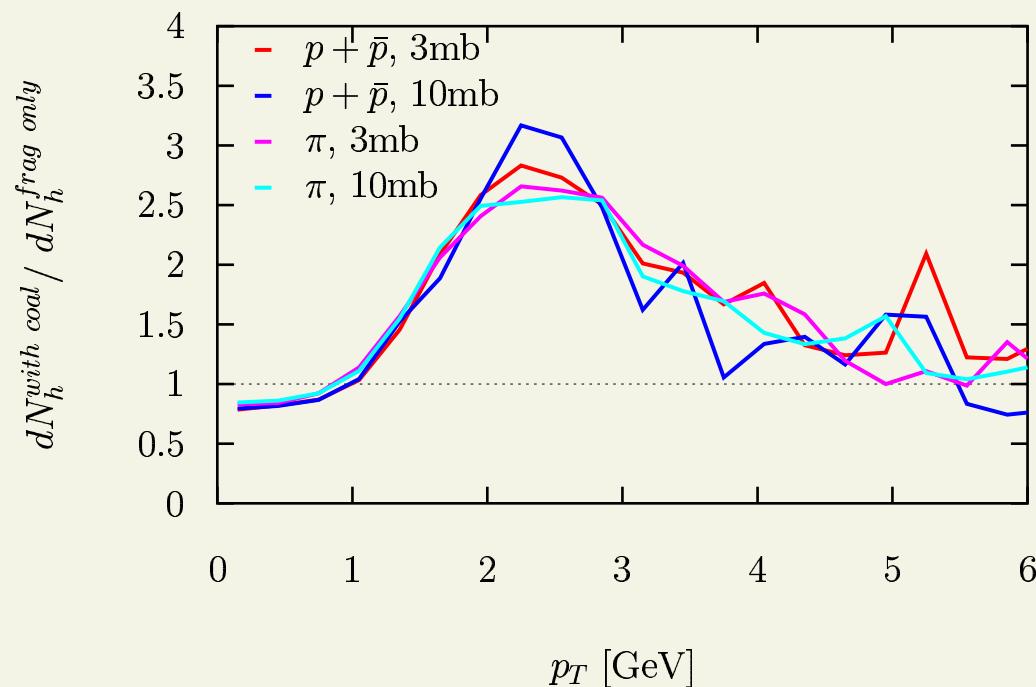
4. spatial dependence can be ignored (factorizes out) $\Rightarrow v_2^{hadron}(p_T) = n v_2^{quark}(p_T/n)$
 - for example, global $v_2(x, p_T) \equiv v_2(p_T)$, or constant FO density

none of these satisfied in transport or hydro, contrary to parameterizations

1. Coal : Frag $\leq 3 : 1$

coal+frag yield / frag only yield

DM ('04): dynamical calculation MPC + coal/JETSET

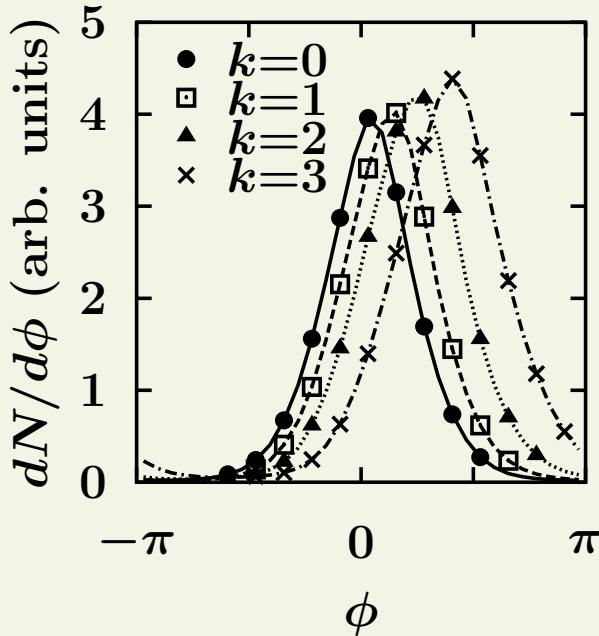
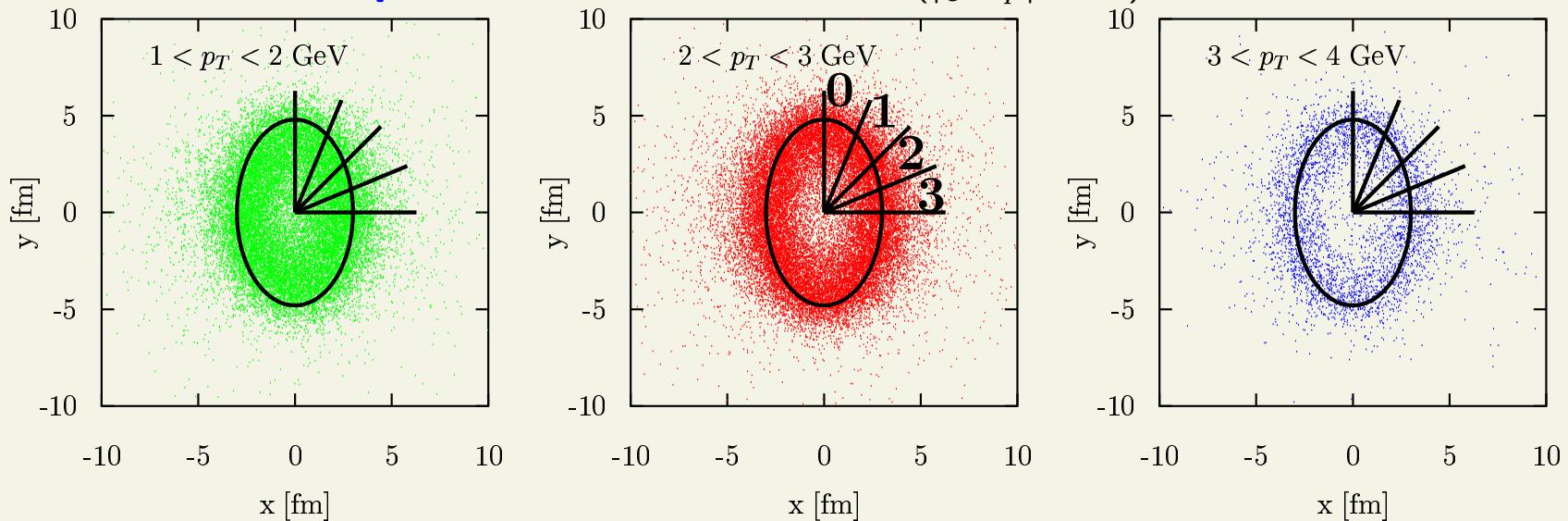


v_2 from $\sim 30\%$ fragmentation contribution does not amplify \rightarrow scaling spoiled

also, about same enhancement for protons and pions $\rightarrow p/\pi$ not enhanced

2. Strong spatial variations

final transverse position distributions ($|y_{rap}| < 2$)



↑ momentum $dN/d\phi$ in each spatial wedge

show surface emission at high p_T $\Rightarrow v_2(x, p_T)$

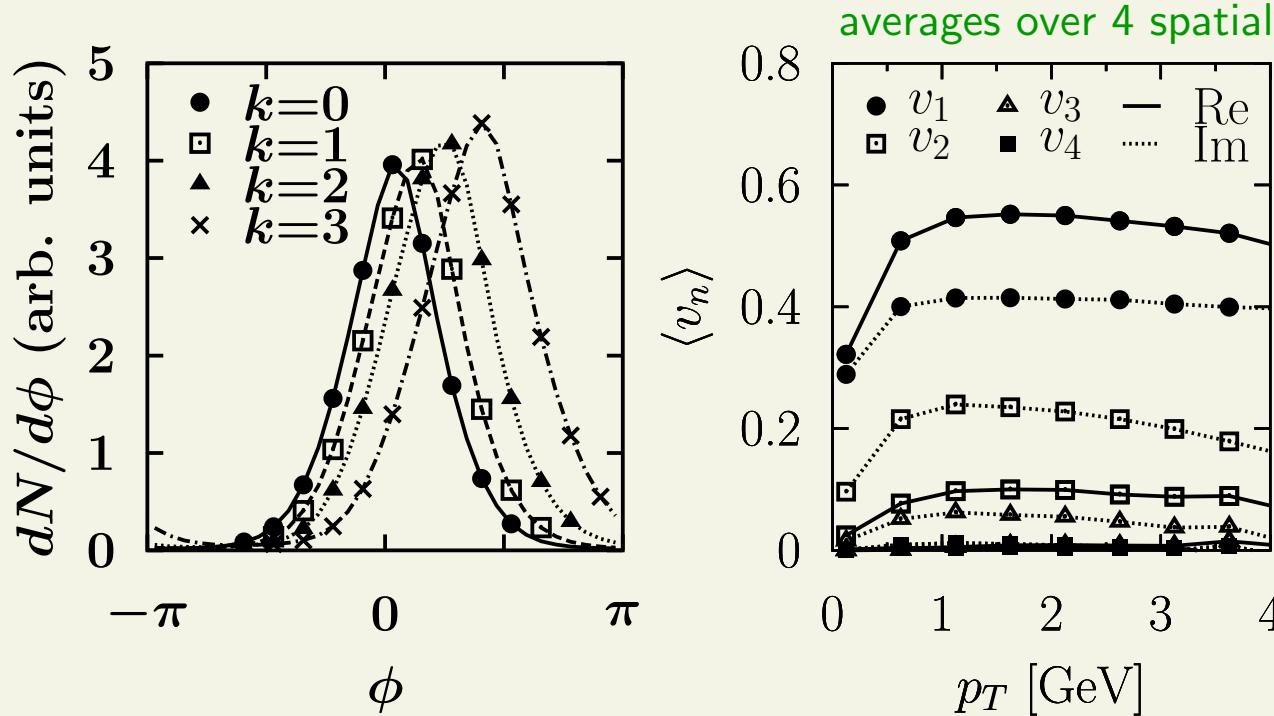
$k = 0$ region: $v_2 < 0$; $k = 3$ region: $v_2 > 0$

expect similar result from hydro

3. Large $|v_n| \sim \mathcal{O}(1)$

DM, nucl-th/0408044

local $\cos(n\phi)$ and $\sin(n\phi)$ anisotropies → use $v_n \equiv \langle \cos(n\phi) + i \sin(n\phi) \rangle$



narrow, almost Gaussian peaks - $dN/d\phi \sim \exp[-(\phi - \phi_0)^2/(2\sigma^2)]$

⇒ $|v_n| \sim \mathcal{O}(1)$, $\langle \cos(n\phi) \rangle \equiv \text{Re}v_n = \cos(n\phi_0) \cdot |v_2| \rightarrow \text{varies with } x(!)$

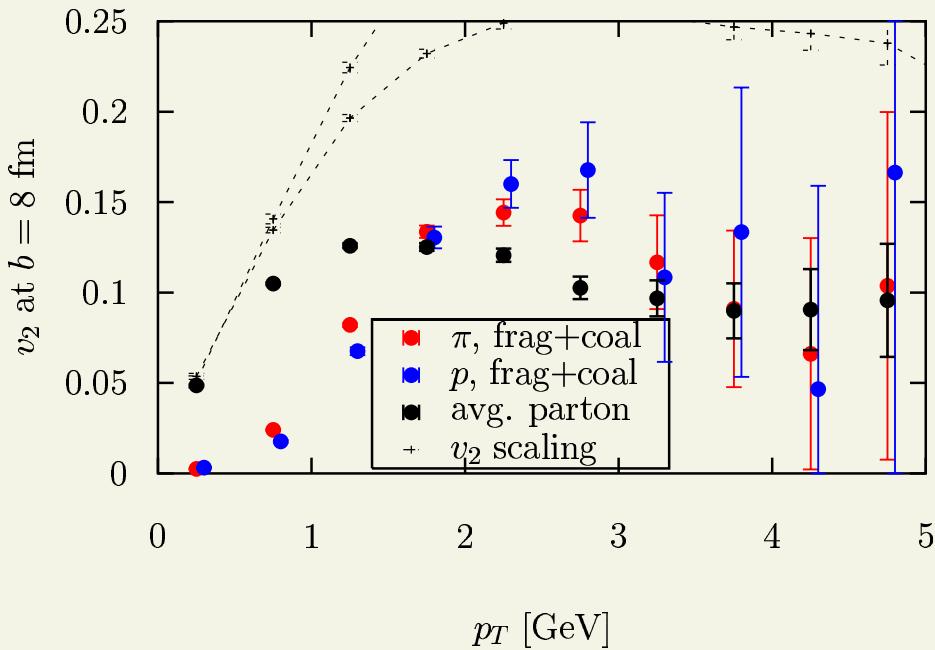
new local scaling: $|v_{k,had}(p_T, x)| \simeq |v_{k,q}(p_T/n_q, x)|^{1/n_q} \neq n_q |v_{k,q}(p_T/n_q, x)|$

Quark number scaling is truly remarkable → PUZZLE #3

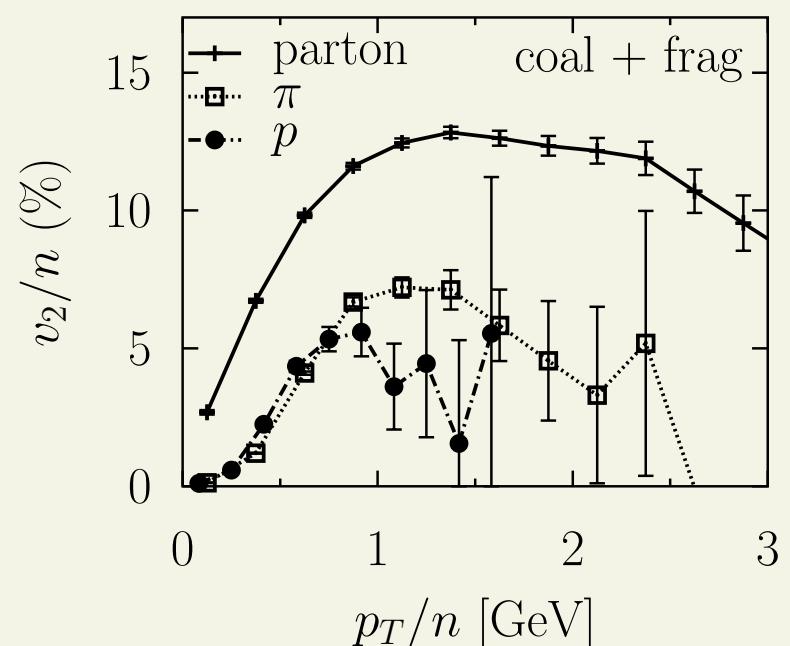
- significant fragmentation contributions
- strong space-momentum correlations (spatial anisotropies)
- surface emission

parton transport + dynamical 4D coalescence - Gyulassy, Frankel, Remler '83
and indep fragmentation - JETSET for partons without coal partner

DM ('04): $v_2(p_T)$ - π, p, q



scaled v_2



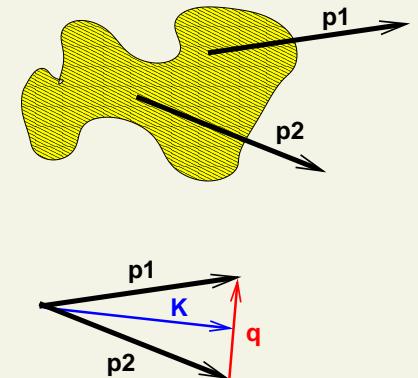
flow amplification greatly reduced, baryon-meson splitting mostly gone

may still scale approximately $\sim 15\%$ err but scaled v_2 is NOT the quark v_2

HBT essentials

Momentum correlations: reflect **spacetime freezeout**

$$C(\vec{q}, \vec{K}) \equiv \frac{N(\vec{p}_1, \vec{p}_2)}{N(\vec{p}_1)N(\vec{p}_2)} \approx 1 + \frac{\left| \int d^4x \ f_{FO}(x, \vec{K}) e^{iq^\mu x_\mu} \right|^2}{\left[\int d^4x \ f_{FO}(x, \vec{K}) \right]^2}$$



[e.g., Pratt, Csörgő & Zimányi, PRC 42, 2646 ('90)]

$f_{FO}(x, \vec{p}) \equiv dN/d^4x \ d^3p$: 7D distribution of **last interaction** vertices

Out-side-long coordinates: special choice of frame

$$K^\mu \equiv (\tilde{K}^0, K_\perp, 0, 0), \quad x^\mu \equiv (\tilde{t}, \textcolor{blue}{x_O}, \textcolor{blue}{x_S}, \textcolor{blue}{x_L}) \quad (\tilde{K}^0 \approx \sqrt{m^2 + K_\perp^2})$$

HBT radii:

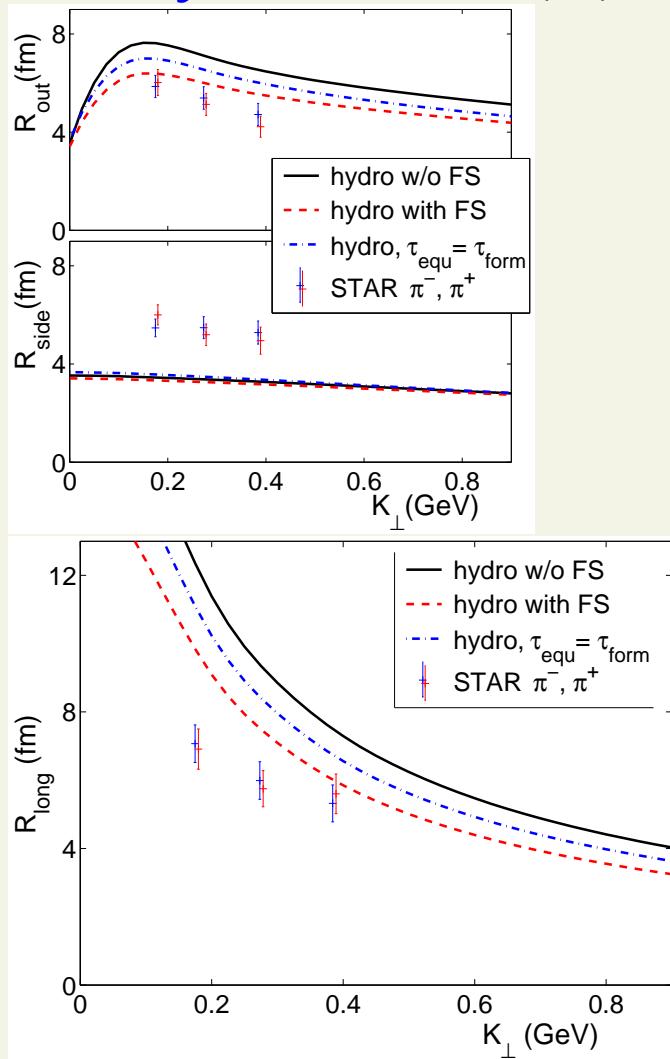
$$R_O^2 = \langle \Delta x_O^2 \rangle_K + v_\perp^2 \langle \Delta \tilde{t}^2 \rangle_K - 2v_\perp \langle \Delta x_O \Delta \tilde{t} \rangle_K$$

$$R_S^2 = \langle \Delta x_S^2 \rangle_K, \quad R_L^2 = \langle \Delta x_L^2 \rangle_K$$

exact for Gaussian source without final-state interactions

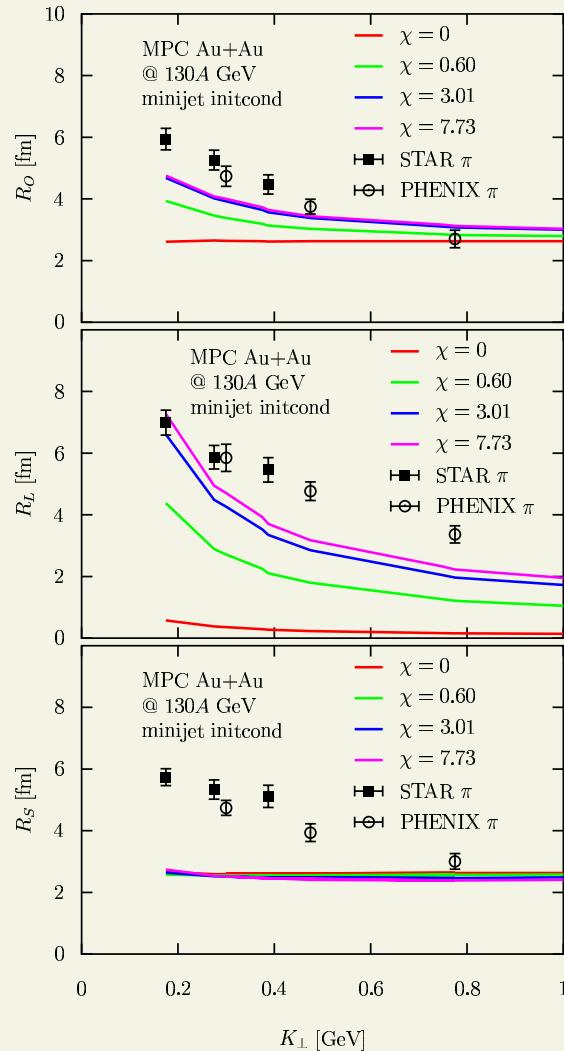
Small R_{side} → PUZZLE # 4

ideal hydro Heinz & Kolb ('02)



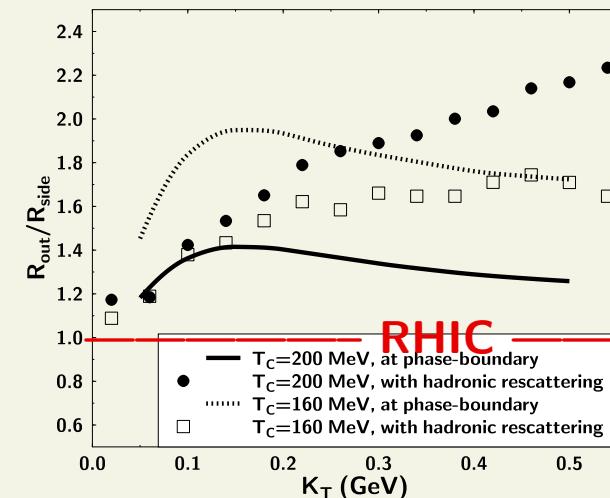
overshoots R_O & R_L
while $R_S \approx 4$ fm only

cov. transport DM & Gyulassy ('02)



R_O & R_L increase with opacity
but $R_S \approx 3.5$ fm stays flat

hydro+transport
Dumitru, Soff ('01)



R_{out}/R_{side} shoots above data

⇒ late-stage hadronic decoupling not understood

wrong spacetime evolution,
or too simple HBT formula?
maybe resonances?

Summary

- Many indications of an opaque, largely randomized (but still dissipative) parton system at RHIC (at 10-100 times the densities of nuclei):
 - strong high-pT suppression of energetic particles
 - large elliptic flow, even for D mesons (prelim.)
 - large baryon/meson ratios, quark number scaling of v_2
 - large “out” and “long” HBT radii
 - this matter seems to be the most ideal fluid ever observed
→ experimental test of minimal viscosity derived from string theory.
 - at such high opacities, soft physics tails can reach up to $p_T \sim 10$ GeV
 - many puzzles and open questions:
 - thermalization mechanism, origin of large opacities
 - R_{AA} vs v_2 opacity inconsistency
 - large charm v_2 but no secondary charm
 - no quark scaling of v_2 & B/M enhancement from dynamical coalescence approach
 - small R_{side} independent of dynamics
 - what will the plasma be like at the LHC (2007)?
- ...